Introduction to EEG/MEG Source Estimation

Olaf Hauk

MRC Cognition and Brain Sciences Unit
olaf.hauk@mrc-cbu.cam.ac.uk

The Basic Problem

What is “the” solution to:

$$x + y = 1$$

If you are not shocked by the EEG/MEG inverse problem…

… then you haven’t understood it yet.

(freely adapted from Niels Bohr)

What Can We Hope For?

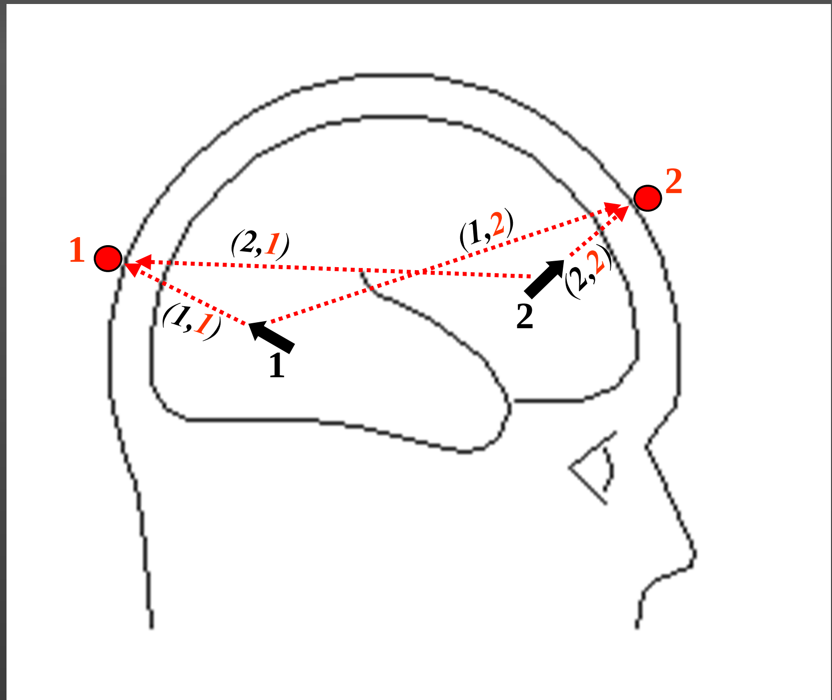
A rough estimate of spatial resolution:

With ***n*** sensors:
-> ***n*** independent measurements
-> at best separate activity from ***n*** brain regions
Sensors are not independent -> ~ 50 degrees of freedom

Volume of source space:
Sphere 8cm minus sphere 4 cm: volume ~5600 cm^3

"Resel": 113 cm^3 -> **4.8**^3 cm^3

Uniquely Solvable Problem



data		"leadfield"		dipoles

$$\begin{pmatrix} d_1 \\ d_2 \end{pmatrix} = \begin{pmatrix} 0.5 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} j_1 \\ j_2 \end{pmatrix}$$

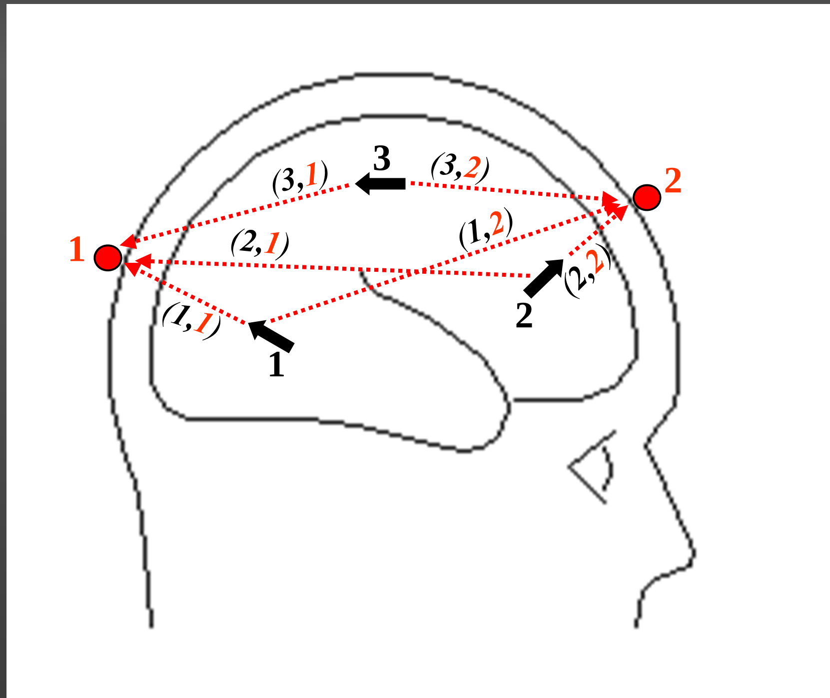
inversion

dipoles		inverse		data

$$\begin{pmatrix} \hat{j}_1 \\ \hat{j}_2 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}$$

Assume dipoles 1 and 2 are only visible to electrodes 1 and 2, respectively.

Non-Uniquely Solvable Problem



“Minimum Norm Solution”

data “leadfield” dipoles

$$\begin{pmatrix} d_1 \\ d_2 \end{pmatrix} = \begin{pmatrix} 0.5 & 0 & 0.3 \\ 0 & 1 & -0.3 \end{pmatrix} \begin{pmatrix} j_1 \\ j_2 \\ j_3 \end{pmatrix}$$

? inversion

dipoles inverse data

$$\begin{pmatrix} j_1 \\ j_2 \\ j_3 \end{pmatrix} = \begin{pmatrix} 1.5034 & 0.1241 \\ 0.2483 & 0.9379 \\ 0.8276 & -0.2069 \end{pmatrix} * \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}$$

Non-Uniqueness

Non-Unique

data "leadfield" dipoles

1 2 1 2 3

$$\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}0.5 & 0 & 0.3\\ 0 & 1 & -0.3\end{pmatrix} * \begin{pmatrix}j_1\\j_2\\j_3\end{pmatrix}$$

?

inversion

"Mininum norm solution:"

dipoles inverse data

1 2 3 1 2

$$\begin{pmatrix}1.62\\1.18\\0.62\end{pmatrix} = \begin{pmatrix}1.5034 & 0.1241\\ 0.2483 & 0.9379\\ 0.8276 & -0.2069\end{pmatrix} * \begin{pmatrix}1\\1\end{pmatrix}$$

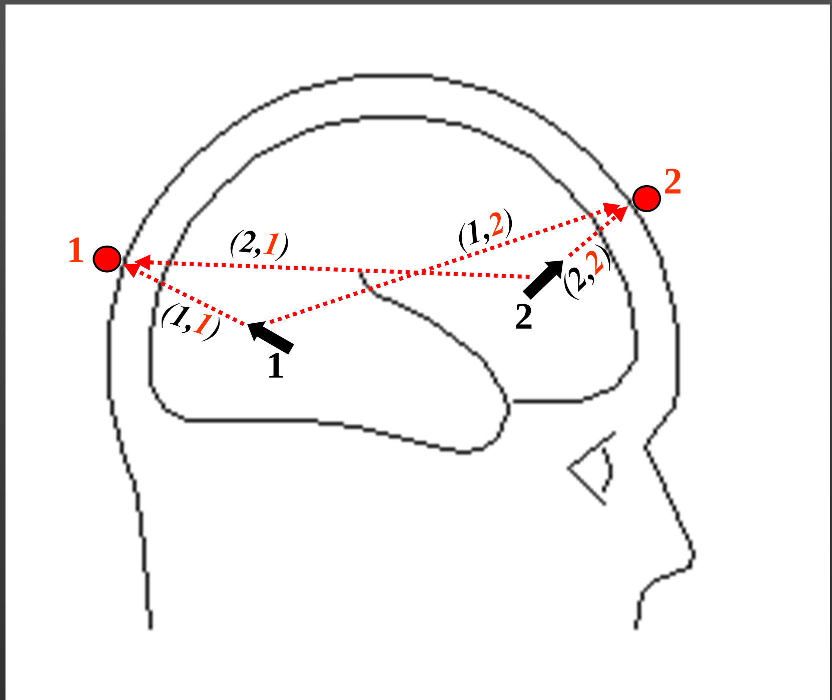
or $\begin{pmatrix}2\\1\\0\end{pmatrix}$ or $\begin{pmatrix}1.81\\1.09\\0.31\end{pmatrix}$

are also possible solutions
that fit the data exactly –

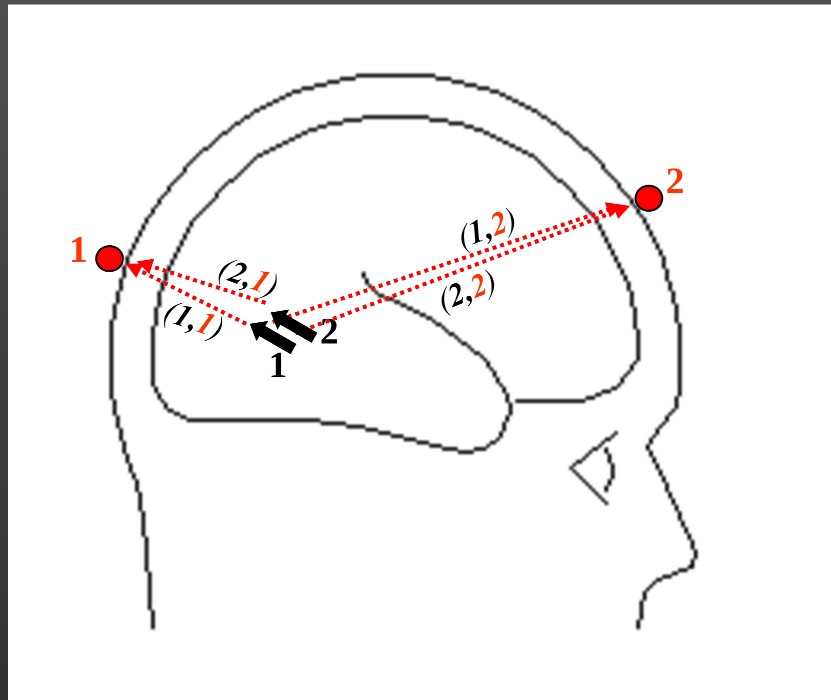
there is no "better" or "worse" solution
Solely on mathematical grounds.

(In)Stability – Sensitivity to Noise

Stable



Instable



(In)Stability – Sensitivity to Noise

Stable

$$\begin{pmatrix} d_1 \\ d_2 \end{pmatrix} = \begin{pmatrix} 0.5 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} j_1 \\ j_2 \end{pmatrix} \quad \xrightarrow{\text{inversion}} \quad \begin{pmatrix} \hat{j}_1 \\ \hat{j}_2 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}$$

Instable

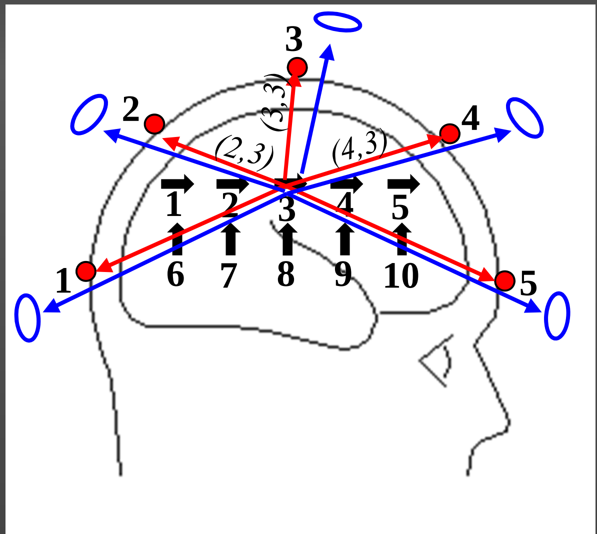
$$\begin{pmatrix} d_1 \\ d_2 \end{pmatrix} = \begin{pmatrix} 0.001 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} j_1 \\ j_2 \end{pmatrix} \quad \xrightarrow{\text{inversion}} \quad \begin{pmatrix} \hat{j}_1 \\ \hat{j}_2 \end{pmatrix} = \begin{pmatrix} 1000 & -1000 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}$$

Addressed by “Regularisation” (“lambda”):

Add smoothness constraint to solution, at the expense of spatial resolution

Recommended to check SNR in source space, “sanity checks”

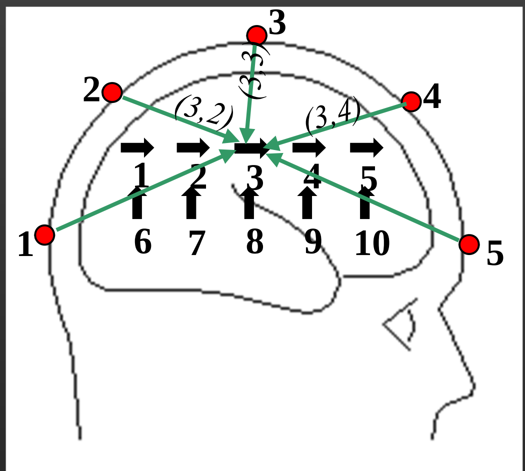
"Forward" and "Inverse" problem



Forward Problem

$$\begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \end{pmatrix} = \mathbf{d} = \sum_{i=1}^{N_s=10} s_i \begin{pmatrix} L_{1i} \\ L_{2i} \\ L_{3i} \\ L_{4i} \\ L_{5i} \end{pmatrix} = \sum_{i=1}^{N_s=10} s_i \mathbf{L}_{.i} = \mathbf{Ls}$$

"Leadfield" matrix
Forward Problem



Inverse Problem

$$\begin{pmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \\ s_6 \\ s_7 \\ s_8 \\ s_9 \\ s_{10} \end{pmatrix} = \mathbf{s} = \sum_{i=1}^{N_c=5} d_i \begin{pmatrix} G_{1i} \\ G_{2i} \\ G_{3i} \\ G_{4i} \\ G_{5i} \\ G_{6i} \\ G_{7i} \\ G_{8i} \\ G_{9i} \\ G_{10i} \end{pmatrix} = \sum_{i=1}^{N_c=5} d_i \mathbf{G}_{.i} = \mathbf{Gd}$$

"Inverse" matrix
"Spatial Filter"

Minimum Norm Estimation: Minimal Modelling Assumptions

"No frills" solution (Minimum Norm)

$$(\hat{\mathbf{s}}-\hat{\mathbf{s}}_0)^T \mathbf{C}_s (\hat{\mathbf{s}}-\hat{\mathbf{s}}_0) = \min$$
$$(\mathbf{L}\hat{\mathbf{s}}-\mathbf{d})^T \mathbf{C}_d (\mathbf{L}\hat{\mathbf{s}}-\mathbf{d}) = \varepsilon > 0$$

⇨ $\hat{\mathbf{s}} = \hat{\mathbf{s}}_0 + \mathbf{C}_s^{-1}\mathbf{L}^T \left(\mathbf{L}\mathbf{C}_s^{-1}\mathbf{L}^T + \lambda\,\mathbf{C}_d^{-1}\right)^{-1}(\mathbf{d}-\mathbf{L}\hat{\mathbf{s}}_0)$ ⇨ $\hat{\mathbf{s}} = \mathbf{L}^T\left(\mathbf{L}\mathbf{L}^T + \lambda\,\mathbf{I}\right)^{-1}\mathbf{d}$

"Most likely" solution (Maximum Likelihood)

$$\mathbf{P}\,(\mathbf{s}) \sim \exp\{-(\hat{\mathbf{s}}-[\mathbf{s}])^T \mathbf{C}_s (\hat{\mathbf{s}}-[\mathbf{s}])\}$$
$$\mathbf{P}\,(\mathbf{d},\hat{\mathbf{s}}) \sim \exp\{-(\mathbf{d}-\mathbf{L}\hat{\mathbf{s}})^T \mathbf{C}_d^{'} (\mathbf{d}-\mathbf{L}\hat{\mathbf{s}})\}$$

⇨ $\hat{\mathbf{s}} = [\mathbf{s}] + \mathbf{C}_s^{-1}\mathbf{L}^T(\mathbf{L}\mathbf{C}_s^{-1}\mathbf{L}^T + \lambda\mathbf{C}_d^{-1})^{-}(\mathbf{d}-\mathbf{L}[\mathbf{s}])$ ⇨ $\hat{\mathbf{s}} = \mathbf{L}^T\left(\mathbf{L}\mathbf{L}^T + \lambda\,\mathbf{I}\right)^{-1}\mathbf{d}$

"Best focussing" solution (Beamformer)

$$Min\,(\mathbf{W}(\mathbf{r}_i - \mathbf{t}_i))^2$$
$$Min([\mathbf{G}_{i.}\mathbf{n}]^2) \Rightarrow Min(\mathbf{G}_{i.}\mathbf{C}_n\mathbf{G}_{i.}^T)$$

⇨ $\mathbf{G}_{i.} = (\mathbf{S} + \lambda\mathbf{C}_n)^{-1}\mathbf{u}$, $\mathbf{S} = \mathbf{L}\mathbf{L}^T$ $\quad \mathbf{u} = \mathbf{L}_{.i}$; $\mathbf{G}_{i.} = (\mathbf{L}\mathbf{L}^T + \lambda\mathbf{I})^{-1}\mathbf{L}_{.i}$ ⇨ $\hat{\mathbf{s}} = \mathbf{L}^T\left(\mathbf{L}\mathbf{L}^T + \lambda\,\mathbf{I}\right)^{-1}\mathbf{d}$

"Minimum Least-Squares Solution"

Under the same modelling assumptions, different approaches converge to the same solution:

"minimum-norm least-squares" (MNLS), "minimum norm estimate" (MNE)

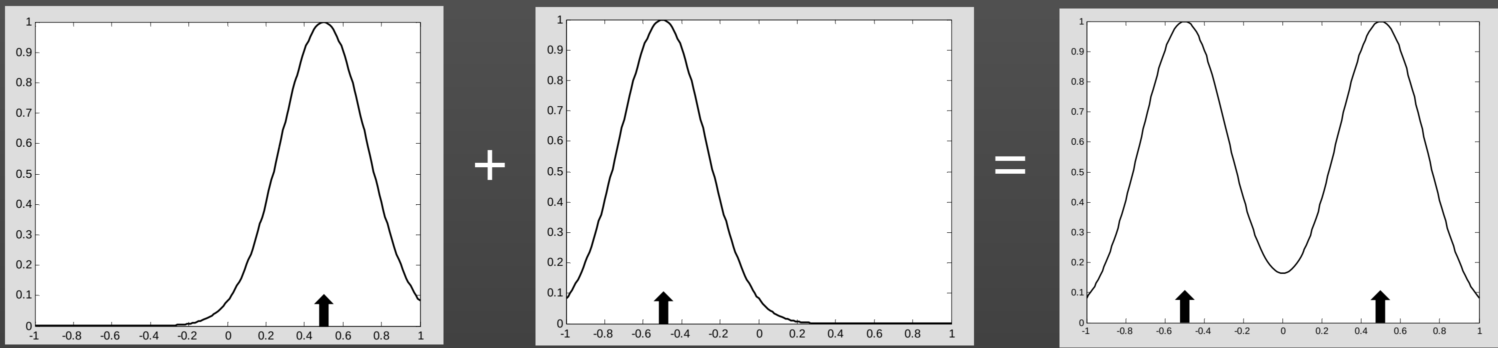
Hauk, Neuroimage 2004

Advantages of Linear Distributed Solutions

Standard in related areas of signal processing and parameter estimation ("General Linear Model")

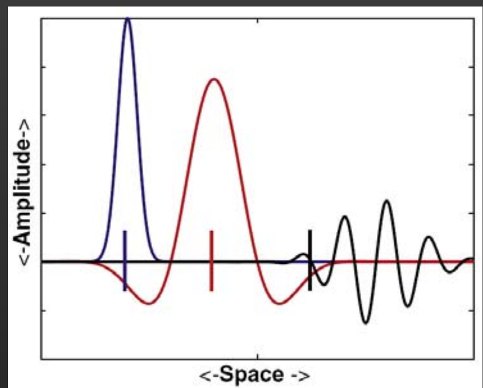
$\Rightarrow$ well-developed theory based on matrix algebra

(Relatively) easy to evaluate, allows generalisable conclusions



The evil you can evaluate is better than the evil you cannot evaluate

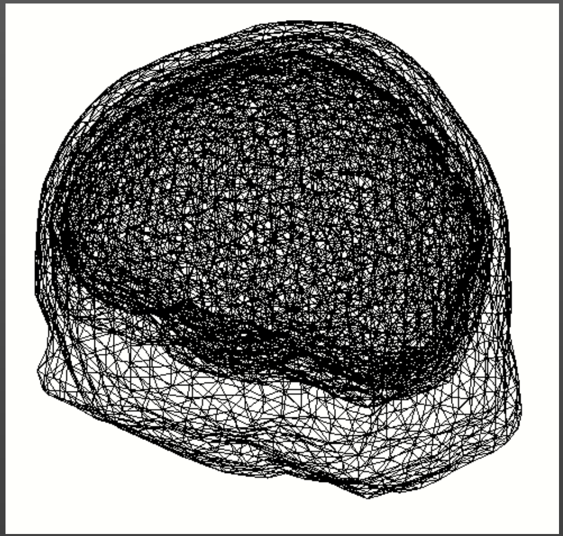
What's the worst than can happen?



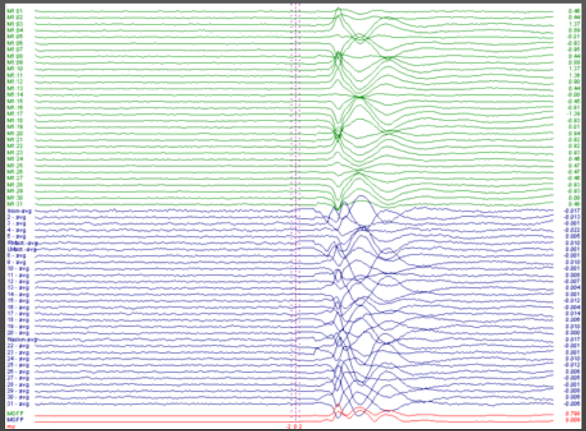
Hauk/Wakeman/Henson, Neuroimage 2011

Ingredients for Source Estimation

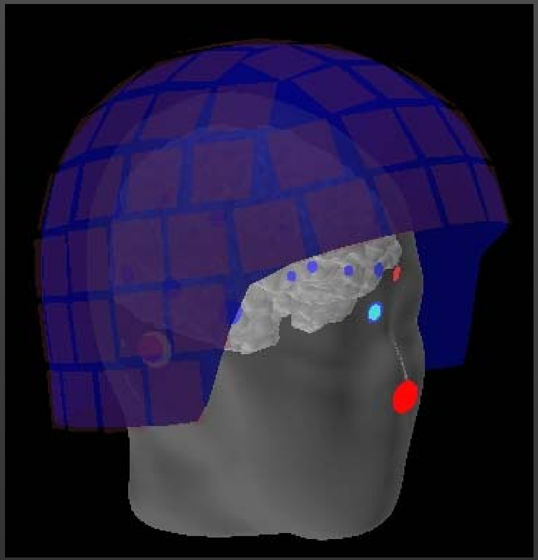
Volume Conductor/
Head Model



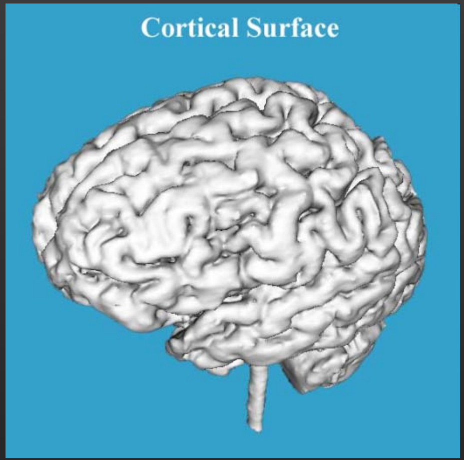
MEG data



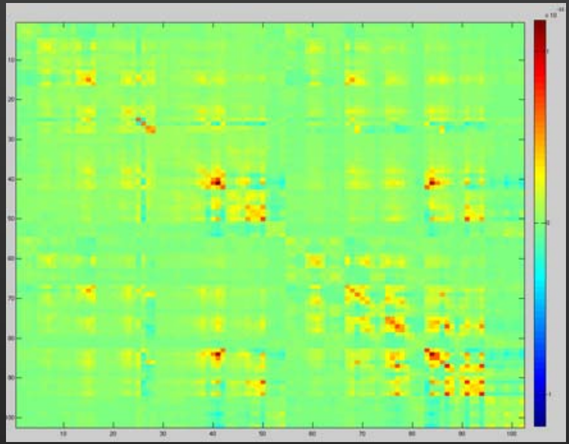
Coordinate
Transformation



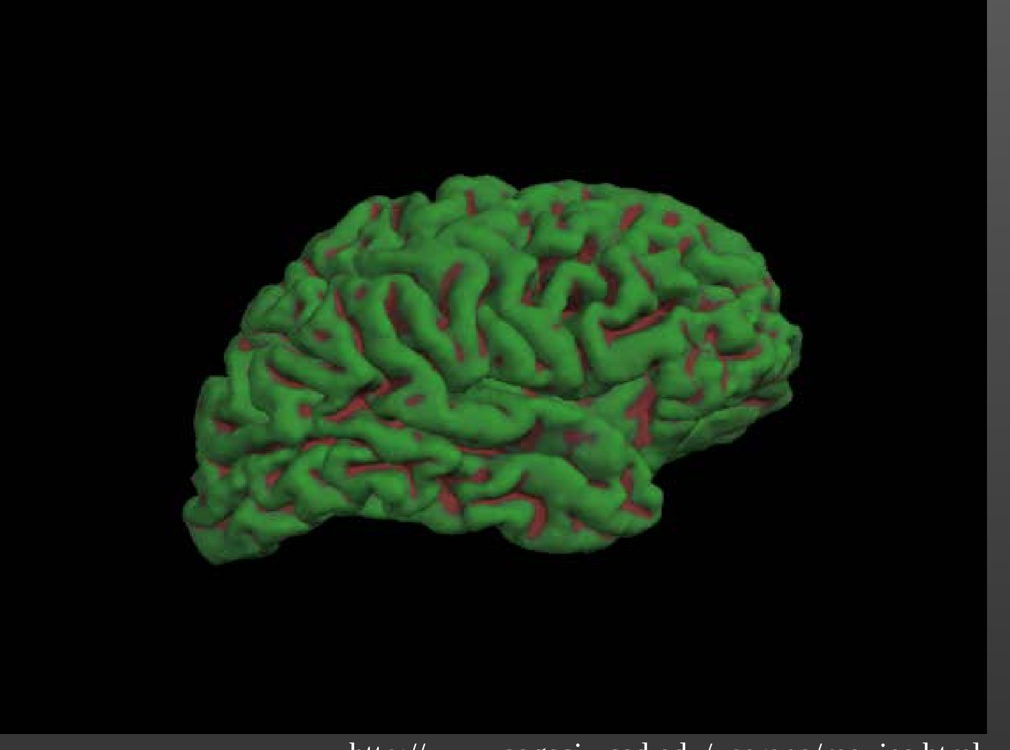
Source Space



Noise/Covariance Matrix



Inflated Cortical Surface



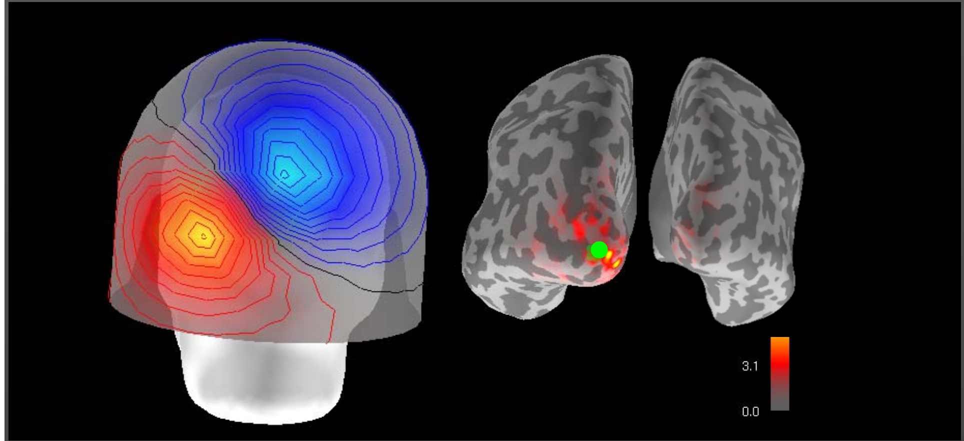
http://www.cogsci.ucsd.edu/~sereno/movies.html

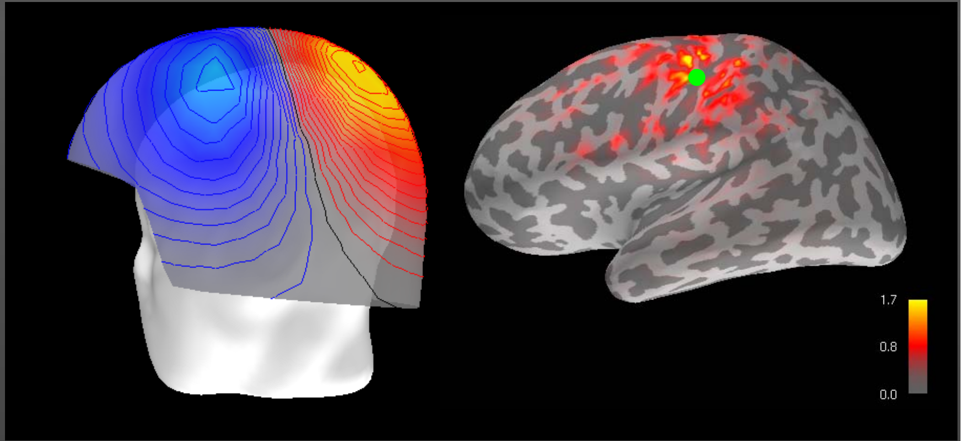
Spatial Resolution of Source Estimation

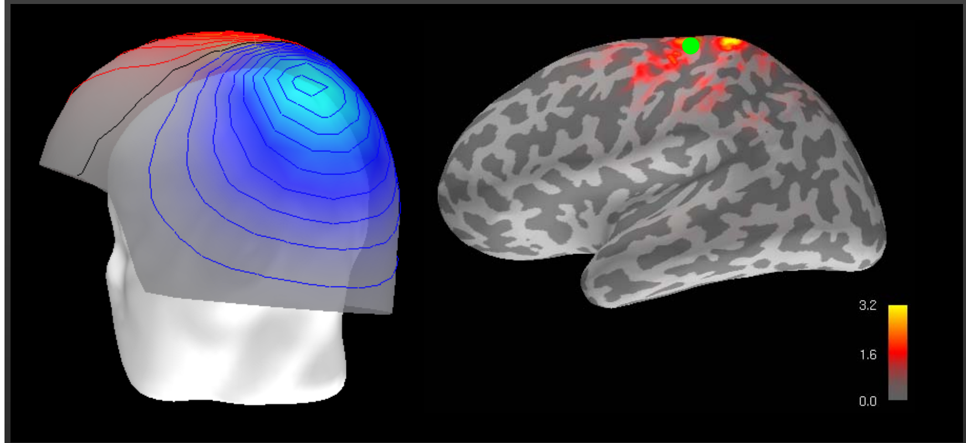
Spatial resolution depends on:

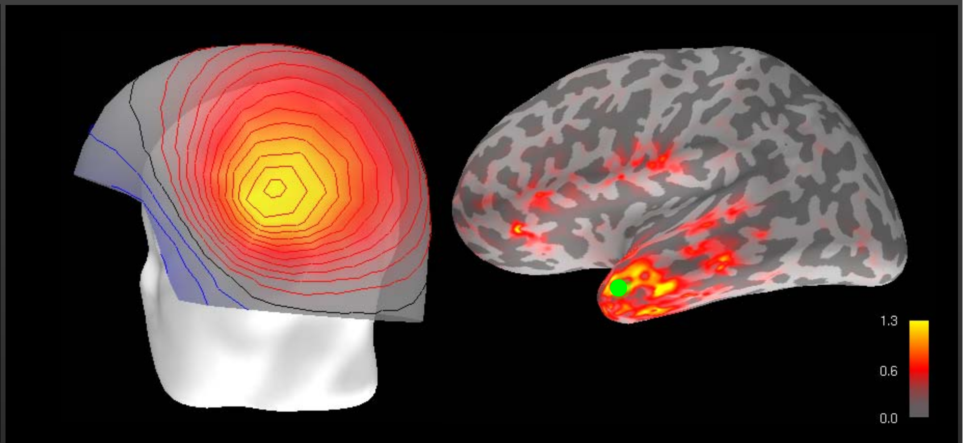
modeling assumptions
number of sensors (EEG/MEG or both)
source location
source orientation
signal-to-noise ratio
head modeling

Localisation for Some ROIs

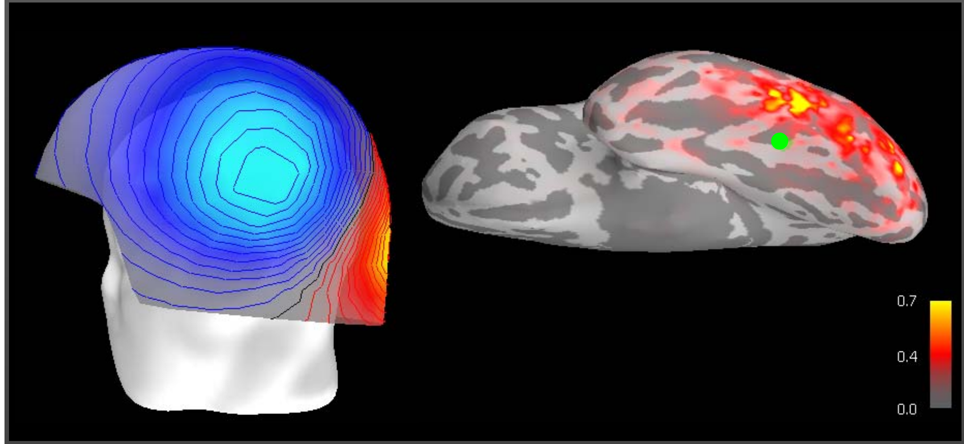


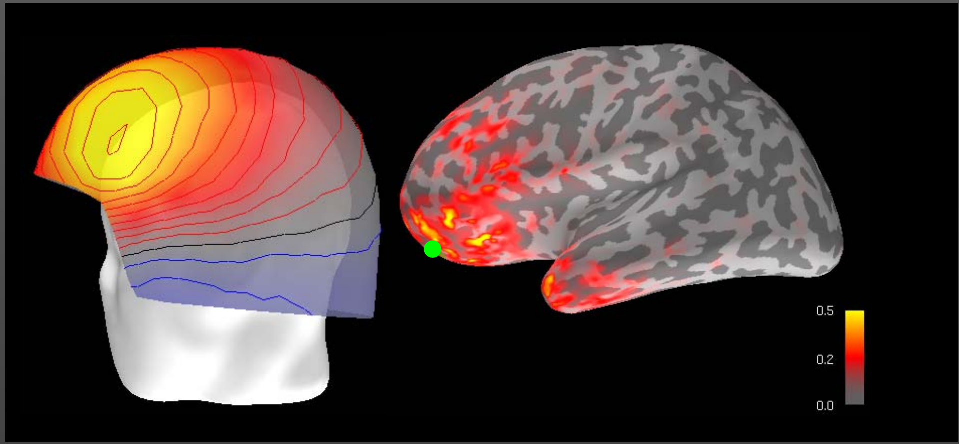


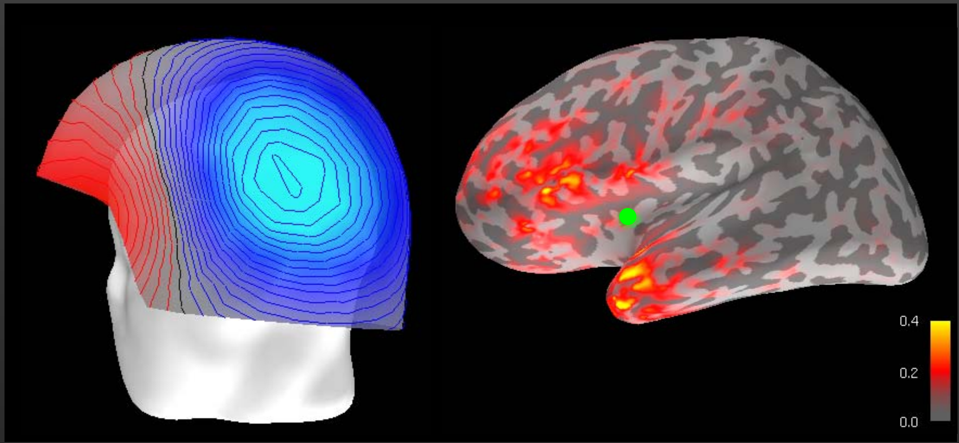




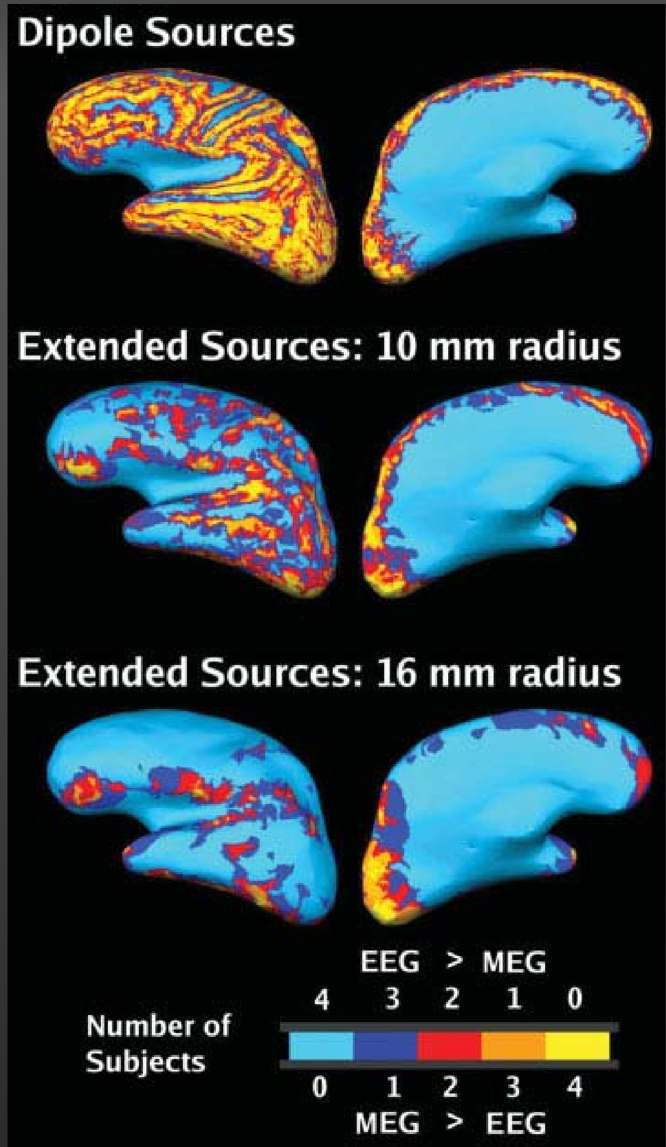
Localisation for Some ROIs







Combining EEG and MEG Increases Sensitivity

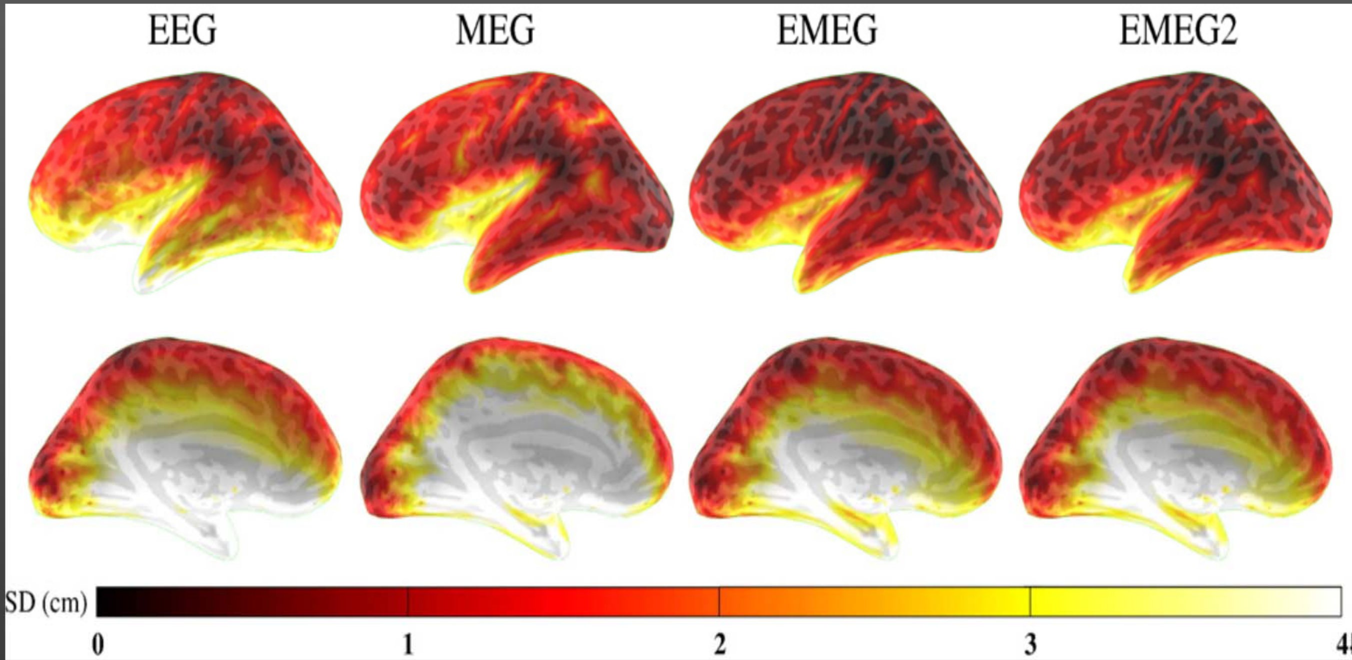


Goldenholz et al., HBM '09

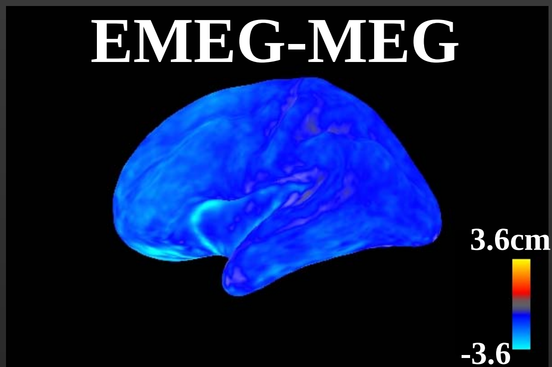
EEG is more sensitive to spatially extended sources

Combining EEG and MEG Improves Resolution

Spatial Extent

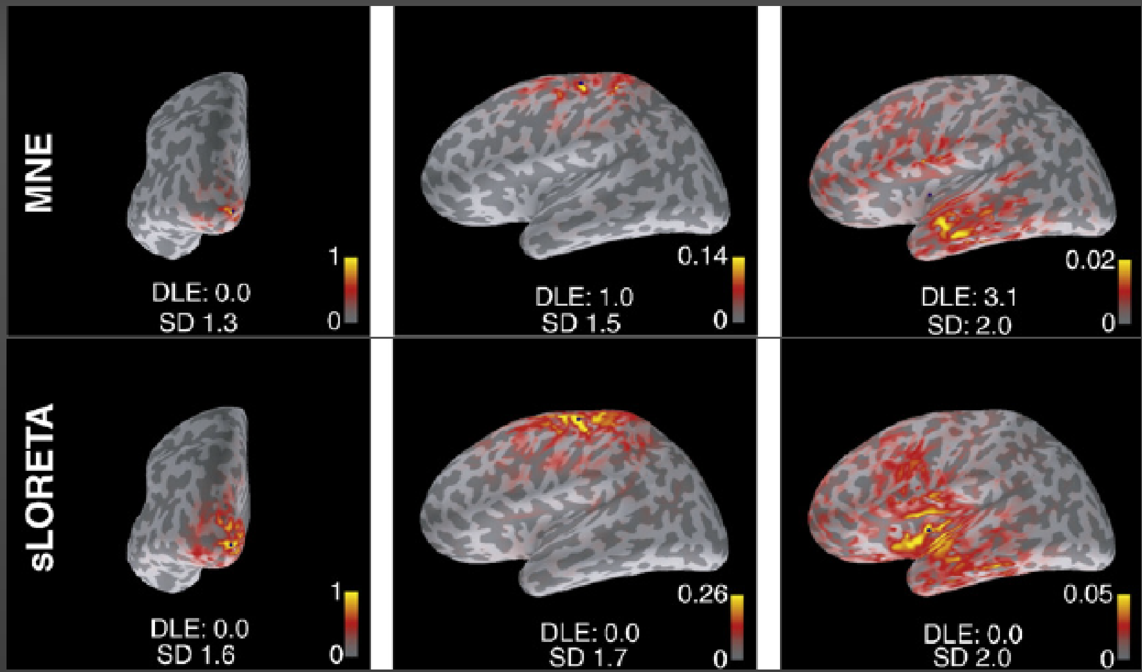


Molins et al., Neuroimage 2008



Stenroos&Hauk, in prep

Source Estimation



"zero dipole localisation error"

Different methods make different compromises

There is no "best" method – best for what?

Hauk/Wakeman/Henson, Neuroimage 2011

Source Estimation Approaches

"Dipole Fitting"

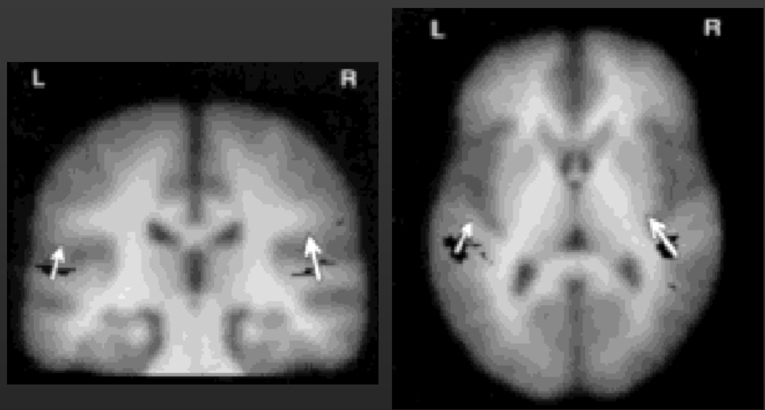
1. Assume there are only a few distinct sources

2. Iteratively adjust the location, orientation and strength of a few dipoles…

3. …until the result best fits the data

Critical parameter:
Residual Variance or Goodness-of-Fit
(ideally taking into account degrees of freedom of the model)

Good for:
Hypothesis testing or precise localisation of well-known sources



Source Estimation Approaches

"Beamforming"

1. Create spatial filter that projects maximally on source of interest…

2. …while minimally projecting on data covariance matrix (incl. signal and noise covariance)

3. "Dipole scan"

No GOF measure

Spatial resolution difficult to evaluate since estimator is data-dependent

Not suitable when source topographies or time courses highly correlated

Applications for spontaneous brain activity (resting state, oscillations), but difficult to justify for evoked responses