



# EEG/MEG Source Estimation Workshop 9 March 2016

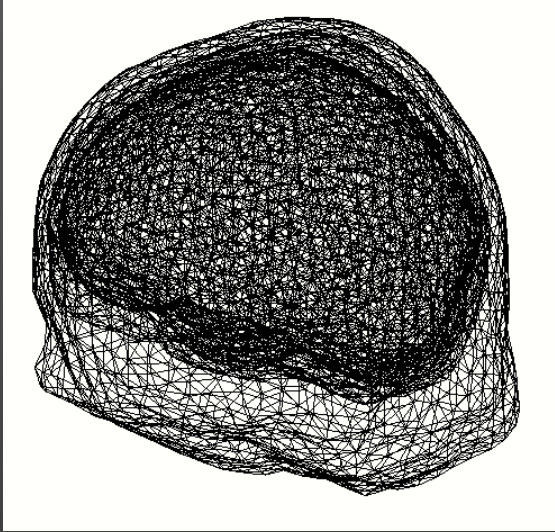
**Olaf Hauk**

**MRC Cognition and Brain Sciences Unit**

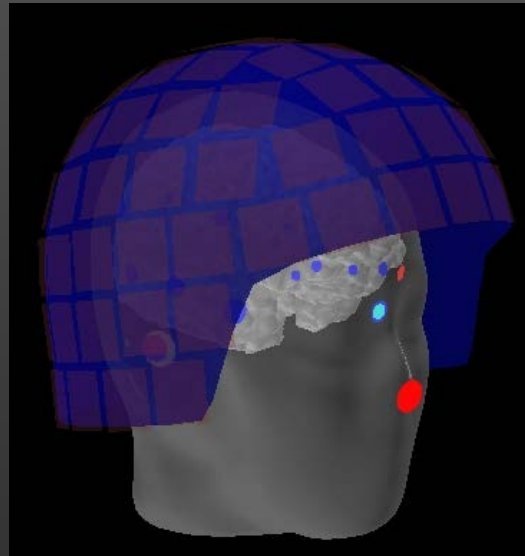
*olaf.hauk@mrc-cbu.cam.ac.uk*

# Ingredients for Source Estimation

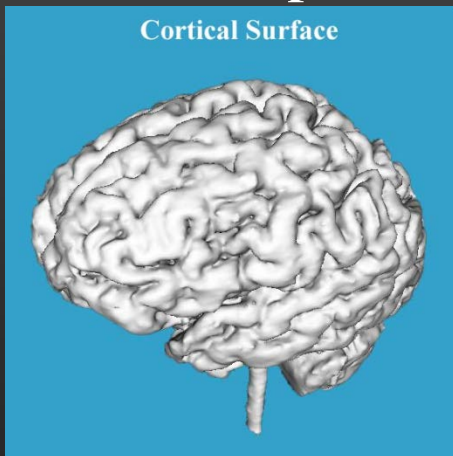
Volume Conductor/  
Head Model



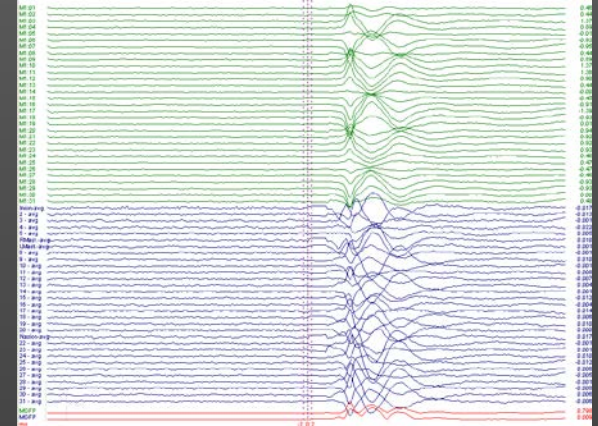
Coordinate  
Transformation



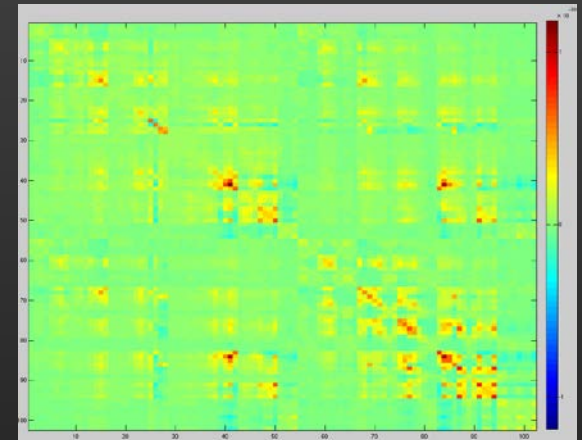
Source Space



MEG data



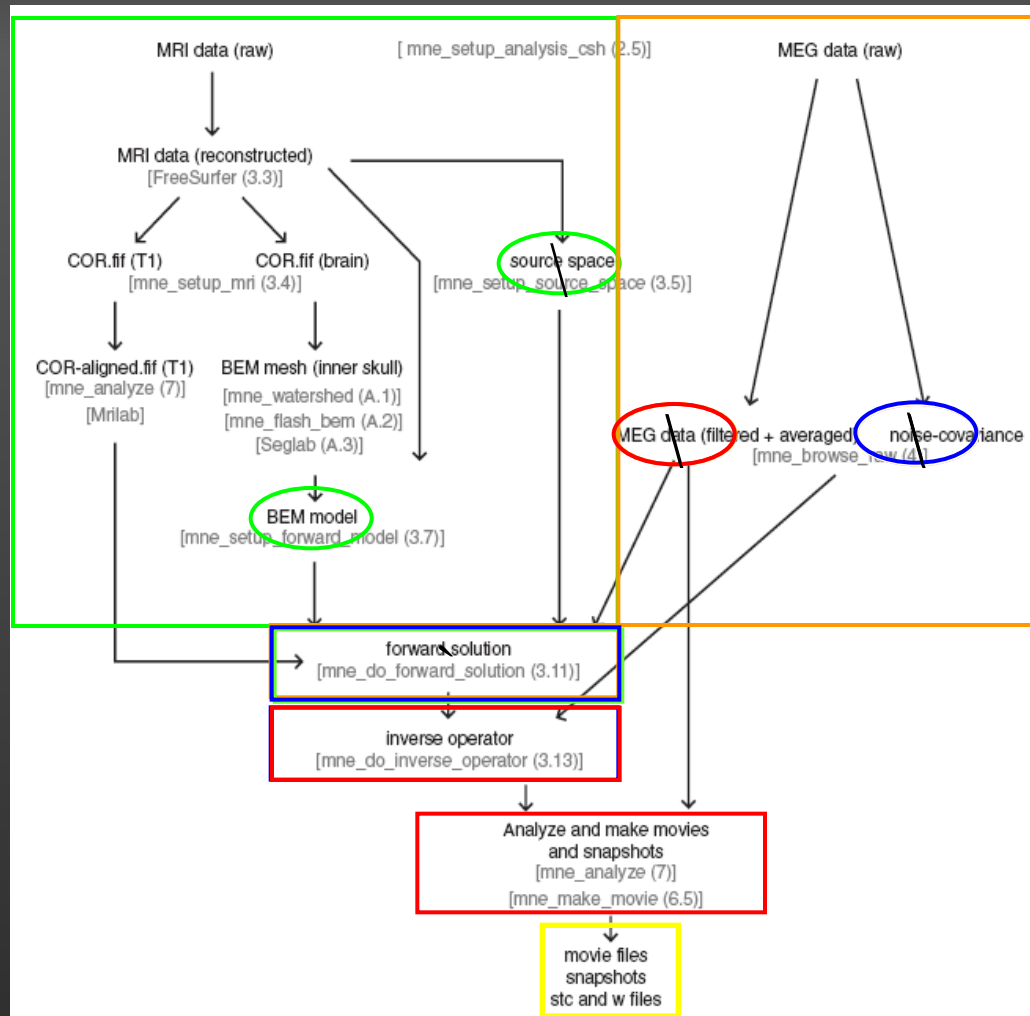
Noise/Covariance Matrix



# The Path to the Source

## MRI

## MEG



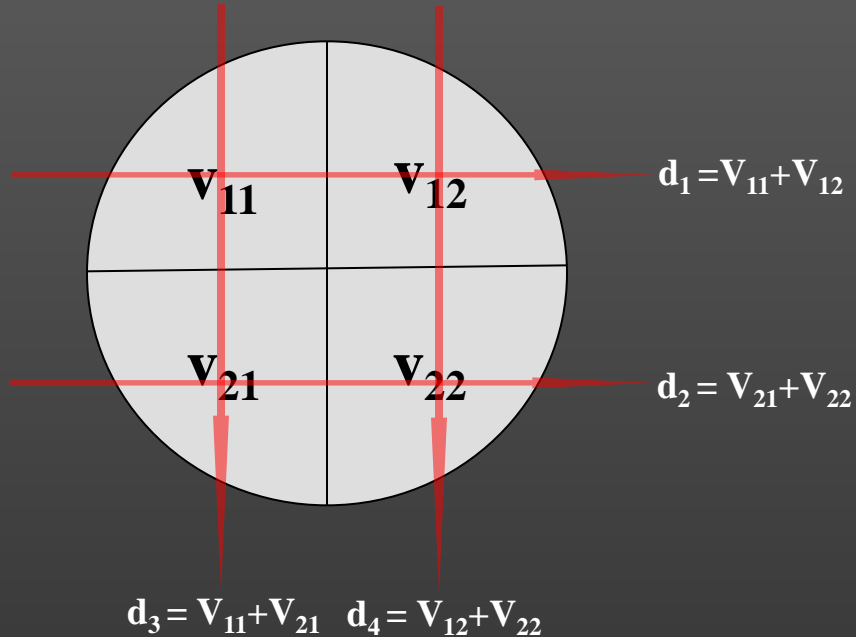
**MNE software:** <http://www.martinos.org/mne/>

See also: <http://www.mrc-cbu.cam.ac.uk/methods-and-resources/imaginganalysis/>



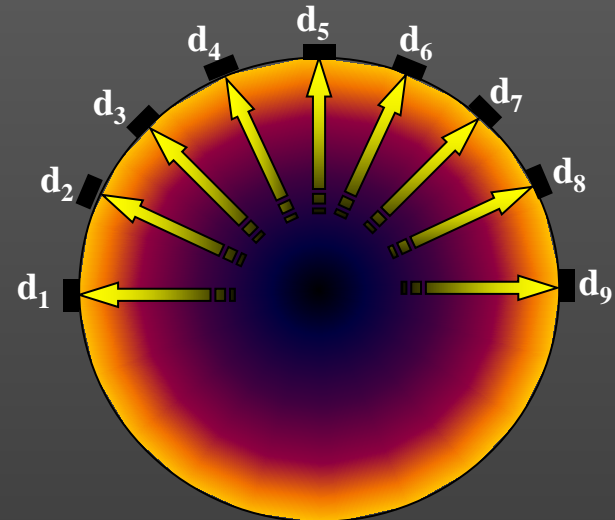
# Why Inverse "Problem"?

## Tomography (CT, fMRI...)



$$\begin{aligned}d_1 &= V_{11} + V_{12} \\d_2 &= V_{21} + V_{22} \\d_3 &= V_{11} + V_{21} \\d_4 &= V_{12} + V_{22}\end{aligned}$$

## EEG/MEG



$$d_1 = V_{11} + V_{12} + V_{13} + V_{14} \dots$$

$$d_2 = V_{21} + V_{22} + V_{23} + V_{24} \dots$$

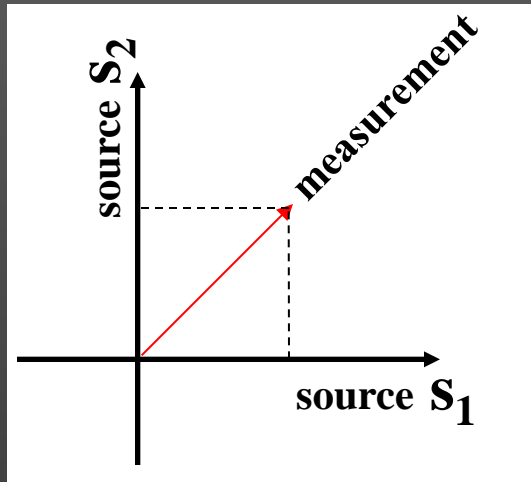
Information is lost during measurement

Cannot be retrieved by mathematics

Inherently limits spatial resolution

# Why Inverse "Problem"?

Reconstructing information  
from an incomplete projection:



We only see a faint shadow of the real distribution of brain activity.

If you are not shocked by the EEG/MEG inverse problem...  
... then you haven't understood it yet.

(freely adapted from Niels Bohr)

# Non-Uniquely Solvable Problem

What is the solution to

$$\mathbf{x}_1 + \mathbf{x}_2 = \mathbf{1}$$

Maybe

$$\mathbf{x}_1 = 0 ; \mathbf{x}_2 = 1 \quad ?$$

$$\mathbf{x}_1 = 1 ; \mathbf{x}_2 = 0 \quad ?$$

$$\mathbf{x}_1 = 1000 ; \mathbf{x}_2 = -999 \quad ?$$

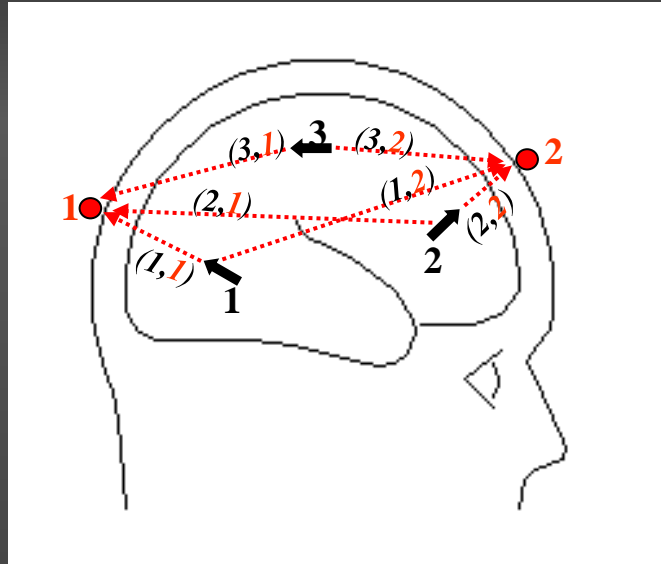
$$\mathbf{x}_1 = \pi ; \mathbf{x}_2 = (1-\pi) \quad ?$$

**The minimum norm solution is:**

$$\mathbf{x}_1 = \mathbf{0.5} ; \mathbf{x}_2 = \mathbf{0.5}$$

with  $(0.5^2 + 0.5^2)=0.5$  the minimum norm among all possible solutions

# Non-Uniquely Solvable Problem



“Minimum Norm Solution”

data	“leadfield”	dipoles	?	dipoles	inverse	data
$\begin{matrix} \bullet \\ \bullet \end{matrix} \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}$	$= \begin{pmatrix} 0.5 & 0 & 0.3 \\ 0 & 1 & -0.3 \end{pmatrix}$	$\begin{pmatrix} j_1 \\ j_2 \\ j_3 \end{pmatrix} \begin{matrix} \swarrow 1 \\ \nearrow 2 \\ \swarrow 3 \end{matrix}$	$\xrightarrow{\text{inversion}}$	$\begin{matrix} \swarrow 1 \\ \nearrow 2 \\ \swarrow 3 \end{matrix} \begin{pmatrix} j_1 \\ j_2 \\ j_3 \end{pmatrix}$	$= \begin{pmatrix} 1.5034 & 0.1241 \\ 0.2483 & 0.9379 \\ 0.8276 & -0.2069 \end{pmatrix}$	$* \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} \begin{matrix} \bullet \\ \bullet \end{matrix}$

MNE produces solution with minimal power or “norm”:

$$\left( j_1^2 + j_2^2 + j_3^2 \right)$$

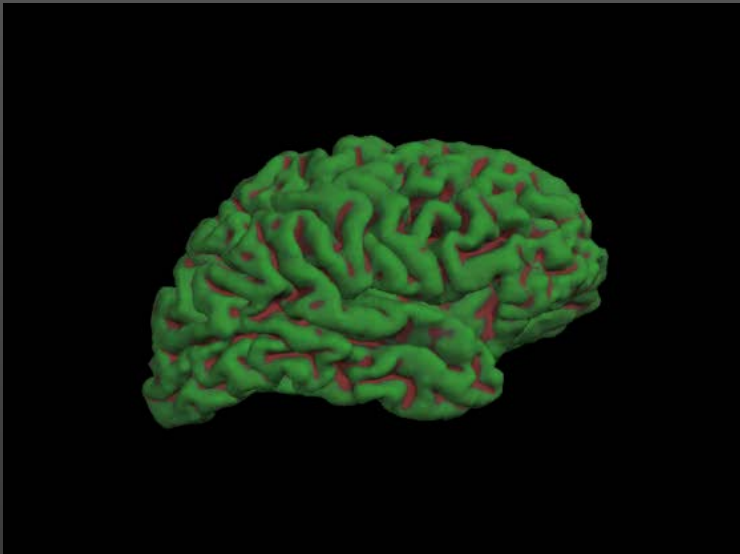




# MRI Preprocessing: Source Space and Head Model

## Source Space,

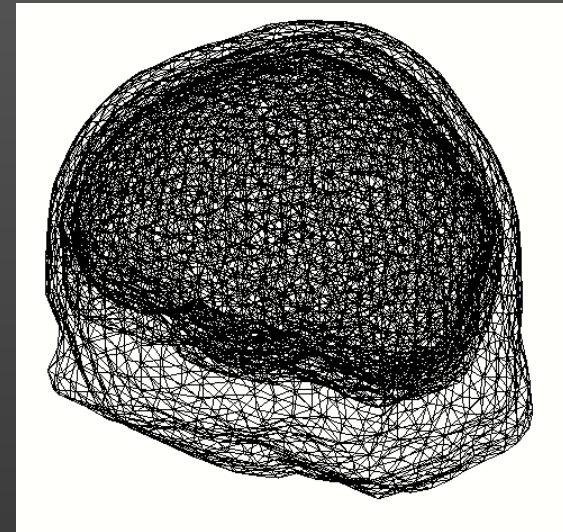
e.g. grey matter, 3D volume



<http://www.cogsci.ucsd.edu/~sereno/movies.html>

## Volume Conductor/Head Model

e.g. sphere, 1- or 3-compartments from MRI

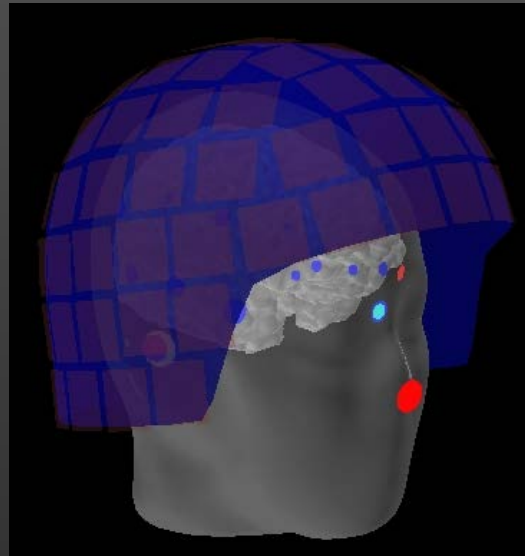


Sometimes “standard head models” are used, when no individual MRIs available.

SPM uses the same “canonical mesh” as source space for every subjects, but adjusts it individually.

# Coregistration of EEG/MEG and MRI Spaces

## Coordinate Transformation

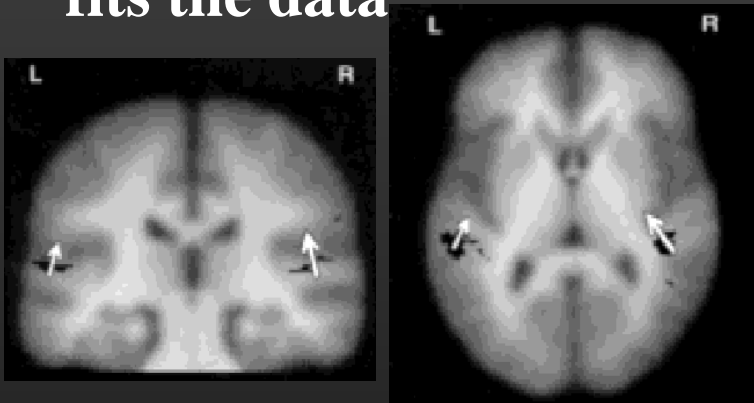




# Source Estimation Approaches

## “Dipole Fitting”

1. Assume there are only a few distinct sources
2. Iteratively adjust the location, orientation and strength of a few dipoles...
3. ...until the result best fits the data



## “Distributed Sources”

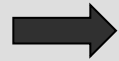
1. Assume sources are everywhere (e.g. distributed across the whole cortex)
2. Find the distribution of source strengths that explains the data...
3. ...AND fulfils other constraints



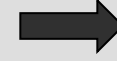
# Minimum Norm Estimation: Minimal Modelling Assumptions

## “No frills” solution (Minimum Norm)

$$\begin{aligned} (\hat{\mathbf{s}} - \hat{\mathbf{s}}_0)^T \mathbf{C}_s (\hat{\mathbf{s}} - \hat{\mathbf{s}}_0) &= \min \\ (\mathbf{L}\hat{\mathbf{s}} - \mathbf{d})^T \mathbf{C}_d (\mathbf{L}\hat{\mathbf{s}} - \mathbf{d}) &= \varepsilon > 0 \end{aligned}$$



$$\hat{\mathbf{s}} = \hat{\mathbf{s}}_0 + \mathbf{C}_s^{-1} \mathbf{L}^T (\mathbf{L} \mathbf{C}_s^{-1} \mathbf{L}^T + \lambda \mathbf{C}_d^{-1})^{-1} (\mathbf{d} - \mathbf{L} \hat{\mathbf{s}}_0)$$

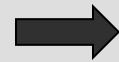


## “Minimum Least-Squares Solution”

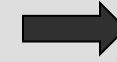
$$\hat{\mathbf{s}} = \mathbf{L}^T (\mathbf{L} \mathbf{L}^T + \lambda \mathbf{I})^{-1} \mathbf{d}$$

## “Most likely” solution (Maximum Likelihood)

$$\begin{aligned} \mathbf{P}(\mathbf{s}) &\sim \exp\{-\hat{\mathbf{s}} - [\mathbf{s}]^T \mathbf{C}_s (\hat{\mathbf{s}} - [\mathbf{s}])\} \\ \mathbf{P}(\mathbf{d}, \hat{\mathbf{s}}) &\sim \exp\{-(\mathbf{d} - \mathbf{L}\hat{\mathbf{s}})^T \mathbf{C}_d (\mathbf{d} - \mathbf{L}\hat{\mathbf{s}})\} \end{aligned}$$



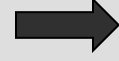
$$\hat{\mathbf{s}} = [\mathbf{s}] + \mathbf{C}_s^{-1} \mathbf{L}^T (\mathbf{L} \mathbf{C}_s^{-1} \mathbf{L}^T + \lambda \mathbf{C}_d^{-1})^{-1} (\mathbf{d} - \mathbf{L}[\mathbf{s}])$$



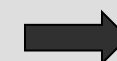
$$\hat{\mathbf{s}} = \mathbf{L}^T (\mathbf{L} \mathbf{L}^T + \lambda \mathbf{I})^{-1} \mathbf{d}$$

## “Best focussing” solution (Beamformer)

$$\begin{aligned} \text{Min}(\mathbf{W}(\mathbf{r}_i - \mathbf{t}_i))^2 \\ \text{Min}([\mathbf{G}_i \mathbf{n}]^2) \Rightarrow \text{Min}(\mathbf{G}_i \mathbf{C}_n \mathbf{G}_i^T) \end{aligned}$$



$$\begin{aligned} \mathbf{G}_i &= (\mathbf{S} + \lambda \mathbf{C}_n)^{-1} \mathbf{u} \\ \mathbf{S} &= \mathbf{L} \mathbf{L}^T \quad \mathbf{u} = \mathbf{L}_i \mathbf{G}_i \end{aligned} \quad \mathbf{G}_i = (\mathbf{L} \mathbf{L}^T + \lambda \mathbf{I})^{-1} \mathbf{L}_i$$



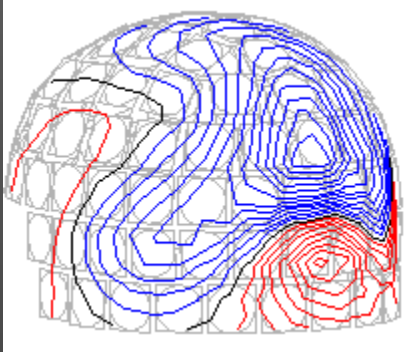
$$\hat{\mathbf{s}} = \mathbf{L}^T (\mathbf{L} \mathbf{L}^T + \lambda \mathbf{I})^{-1} \mathbf{d}$$



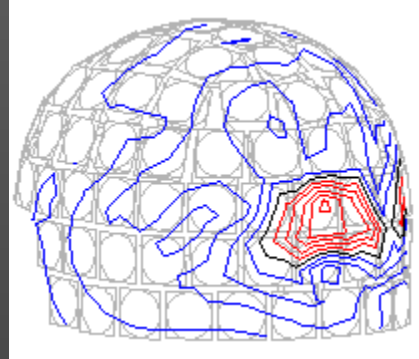
All approaches converge to the same solution if no a priori information is available

There are many possible assumptions, and therefore many different methods – but unfortunately no gold standard to properly compare them

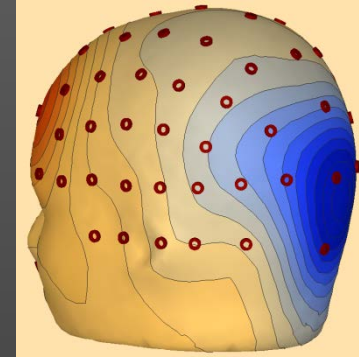
# Visually Evoked Activity ~100 ms



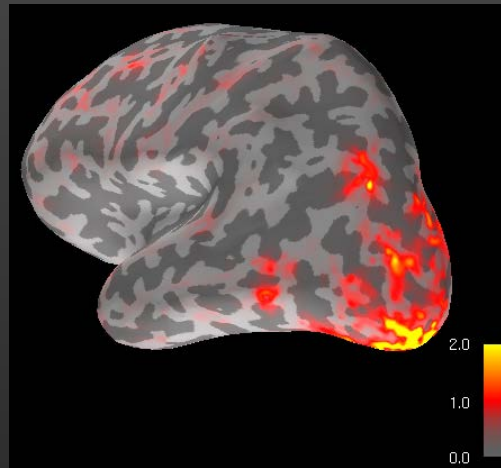
Magnetometers



Gradiometers

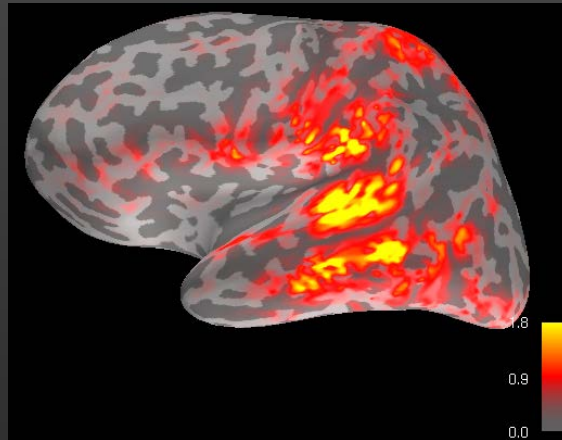
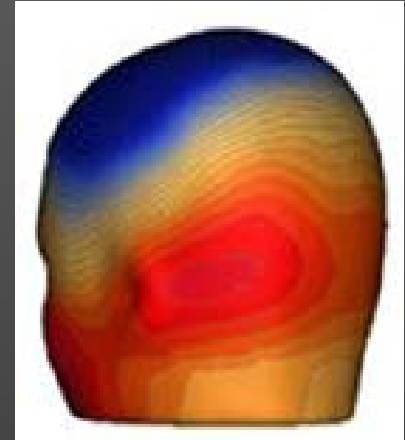
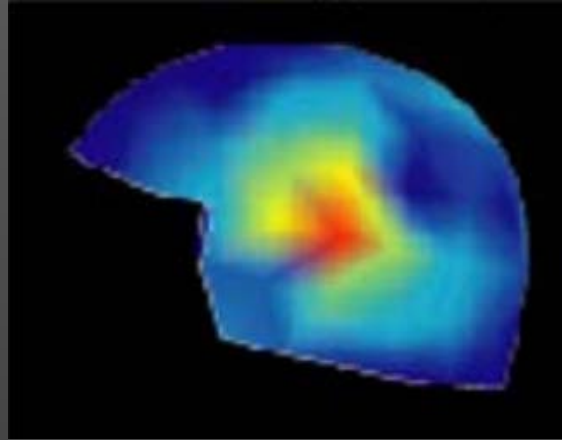
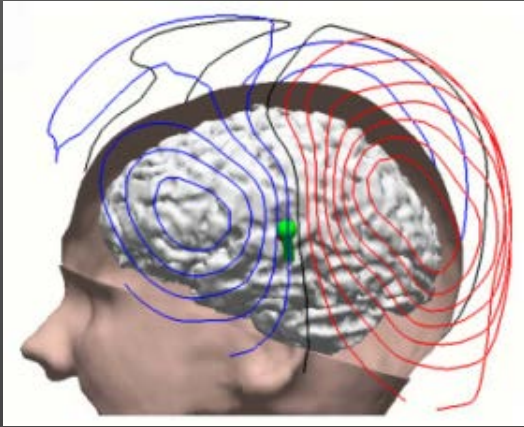


EEG



Minimum Norm Estimate

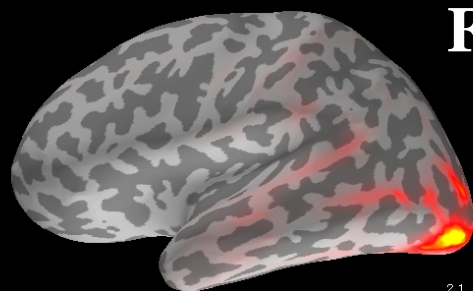
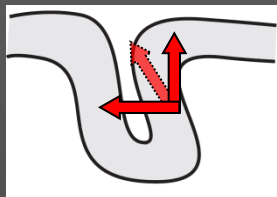
# Auditorily Evoked Activity



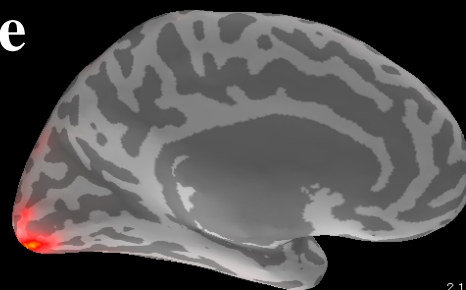
**Minimum Norm Estimate**



# Source Orientation Constraints



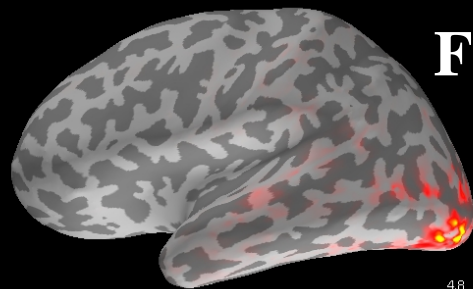
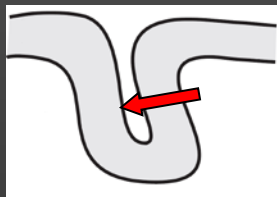
**Free**



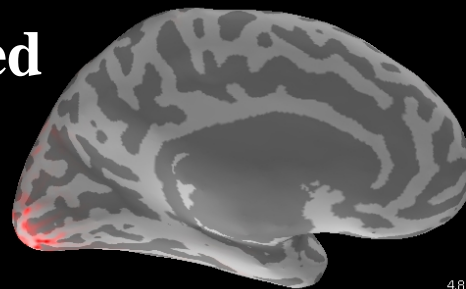
MNE : 003-loose  
108.00 ms  
0.00 .. 1.05 .. 2.1 \* 1e-10



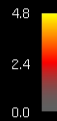
MNE : 003-loose  
108.00 ms  
0.00 .. 1.05 .. 2.1 \* 1e-10



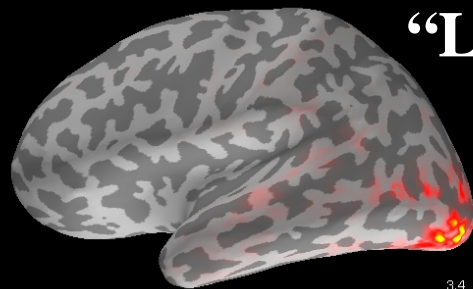
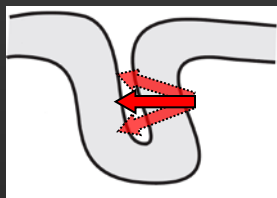
**Fixed**



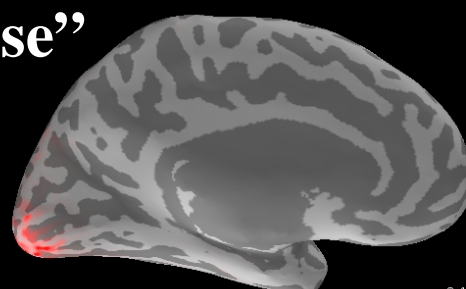
MNE : 003-fixed  
108.00 ms  
0.00 .. 2.40 .. 4.8 \* 1e-10



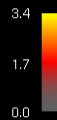
MNE : 003-fixed  
108.00 ms  
0.00 .. 2.40 .. 4.8 \* 1e-10



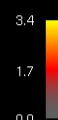
**“Loose”**



MNE : 003-loose02  
108.00 ms  
0.00 .. 1.69 .. 3.4 \* 1e-10

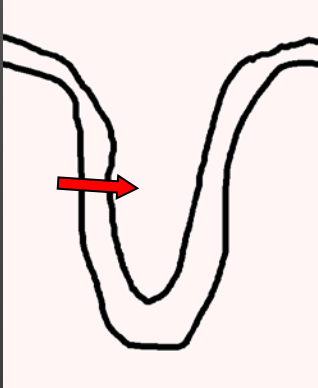


MNE : 003-loose02  
108.00 ms  
0.00 .. 1.69 .. 3.4 \* 1e-10

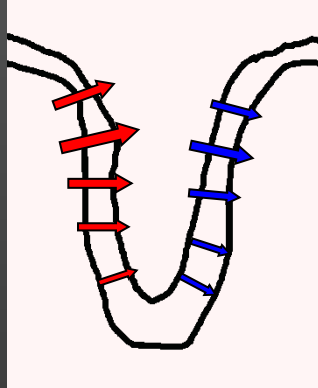


# Direction of Current Flow

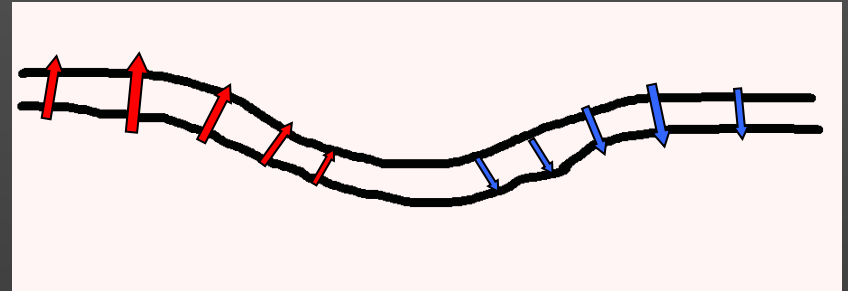
Dipole Source



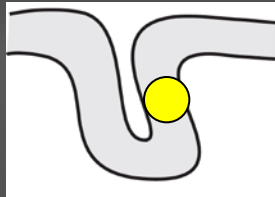
Distributed Source



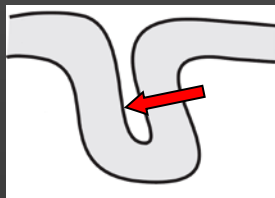
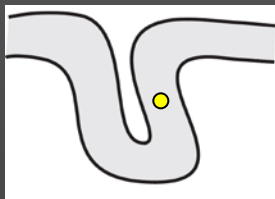
Distributed Source, Inflated Surface



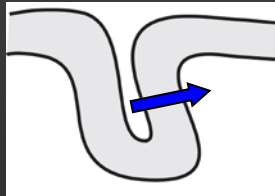
# Direction of Current Flow



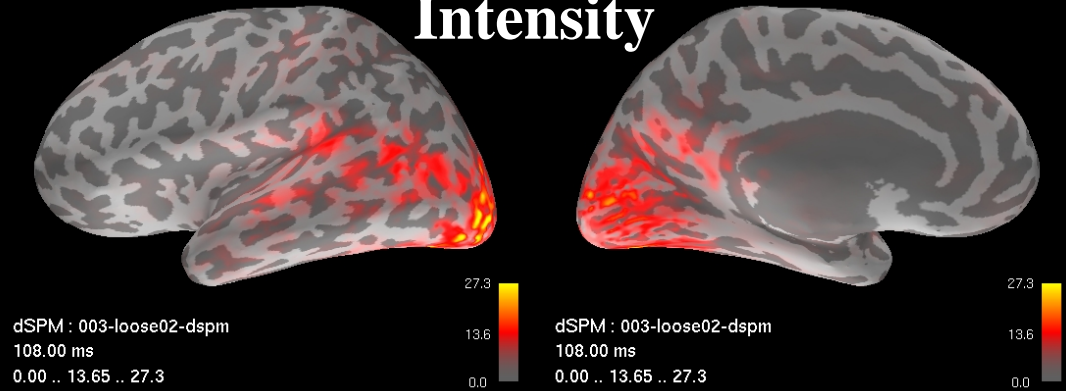
or



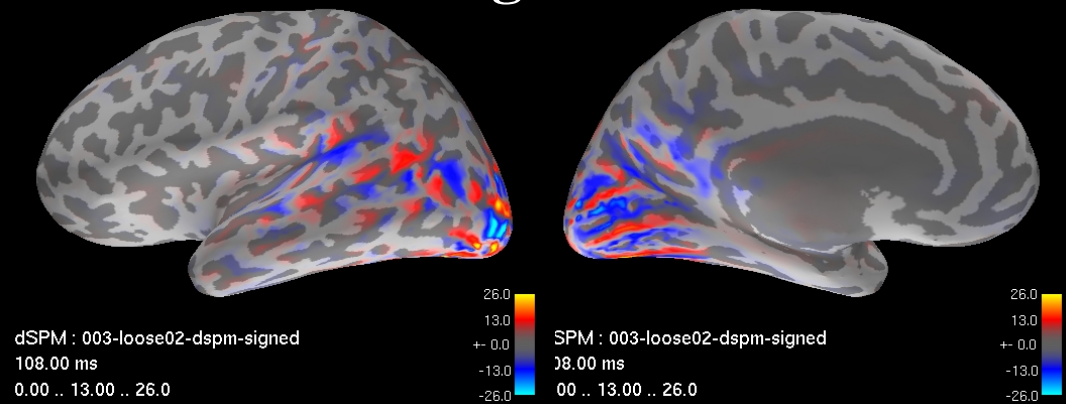
or



## Intensity



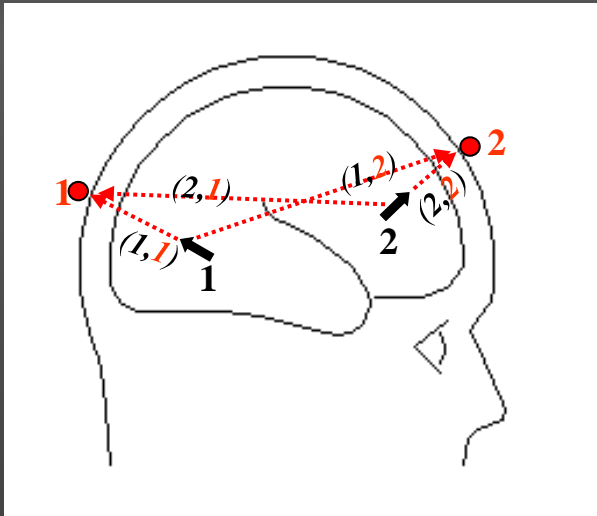
## “signed”



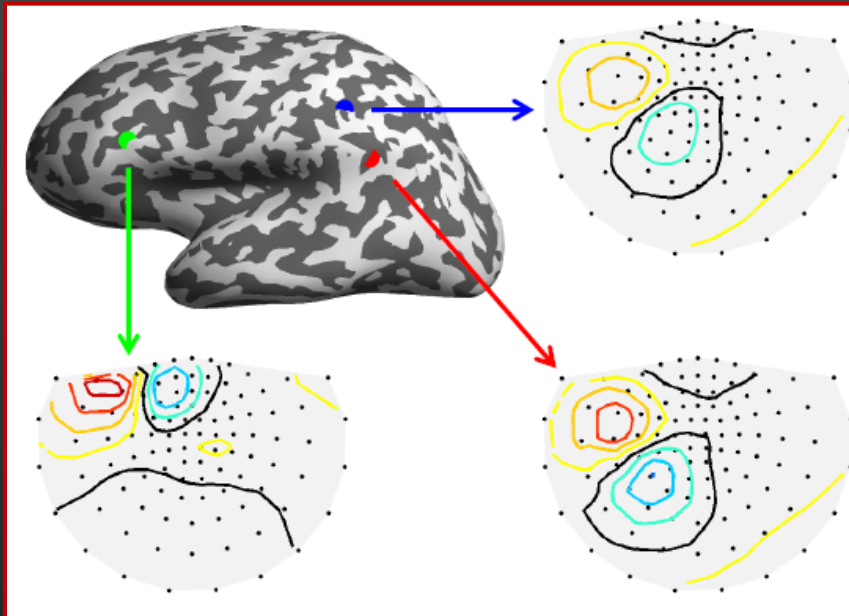
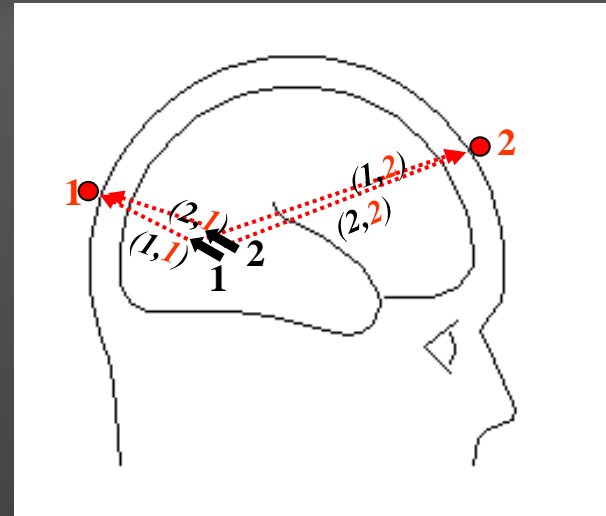


# (In)Stability - Sensitivity to Noise

## Stable



## Instable



Similar topographies are difficult to distinguish, especially in the presence of noise.

# Noise covariance

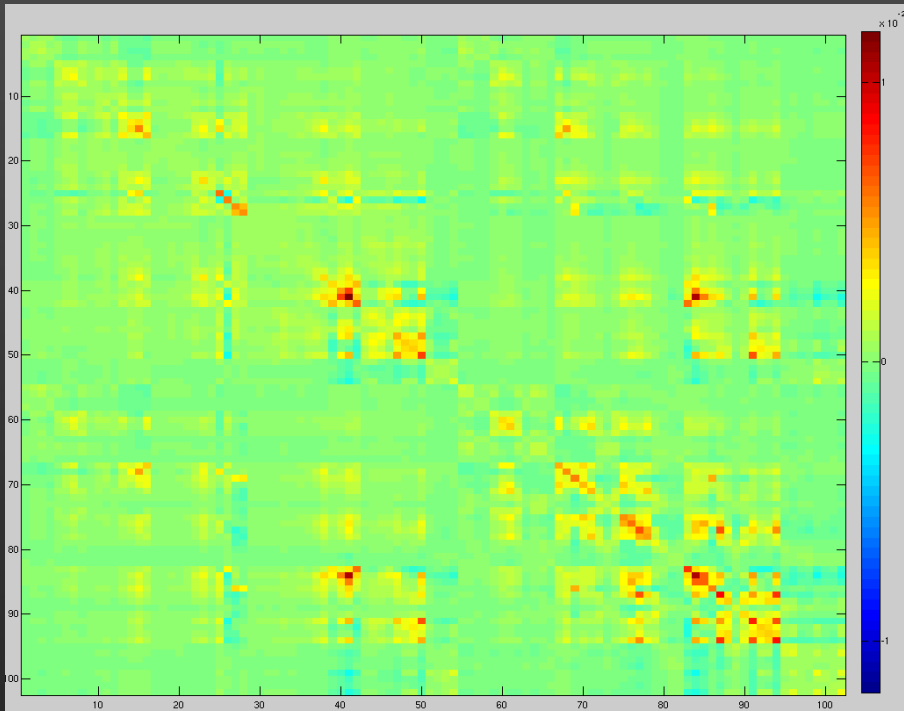
Some channels are noisier than others

⇒ They should get different weights in your analysis

Sensors are not independent

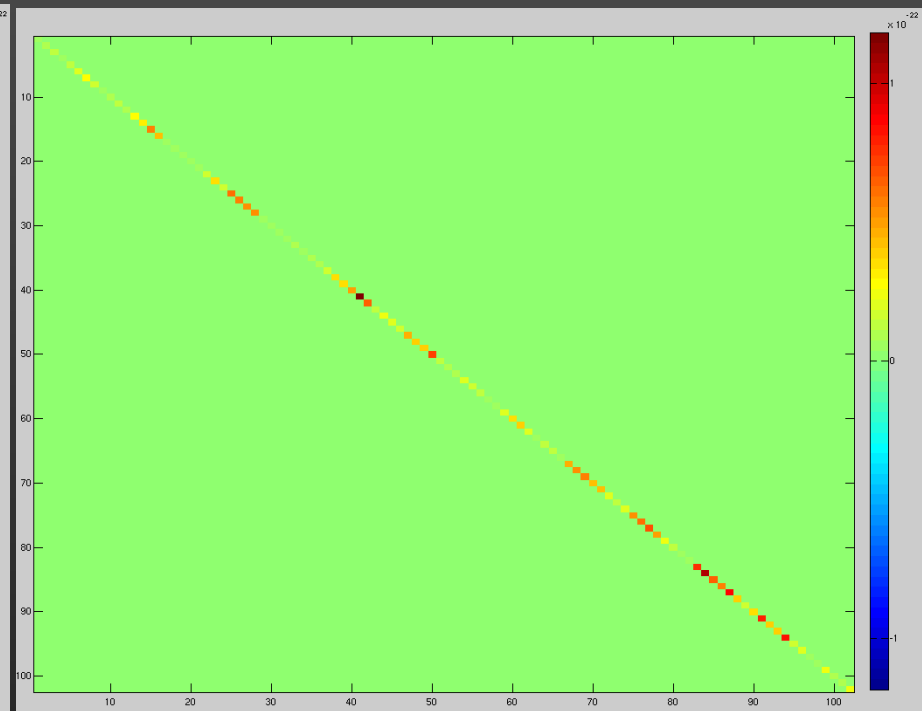
⇒ Sensors that carry the same information should be downweighted relative to more independent sensors

**(Full) Noise Covariance Matrix**



**(Diagonal) Noise Covariance Matrix**

(contains only variance for sensors)

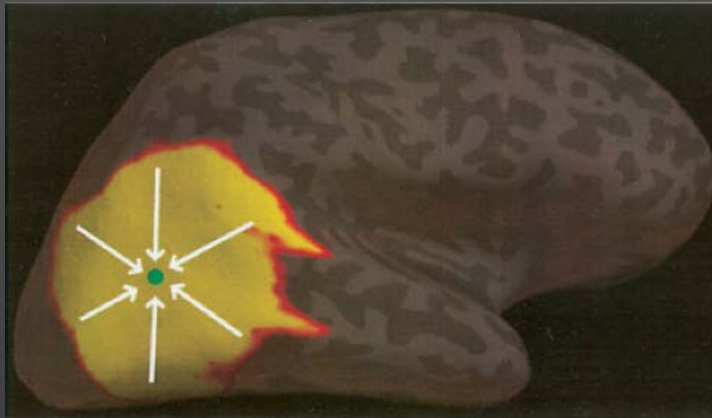




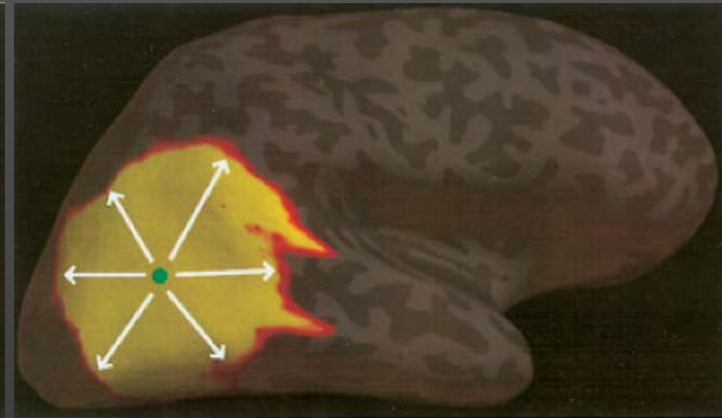
# Spatial Resolution:

## Point-Spread and “Cross-Talk”/“Leakage”

### Cross-Talk/Leakage



### Point-Spread



Liu et al., HBM 2002

*“How other sources may affect the spatial filter for this source”*

*“How this source affects other spatial filters”*



# Spatial Resolution of Source Estimation

**Spatial resolution depends on:**

**modeling assumptions**

**number of sensors (EEG/MEG or both)**

**source location**

**source orientation**

**signal-to-noise ratio**

**head modeling**

**=> difficult to make general statement**

# Spatial Resolution - A Naïve Estimate

With  $n$  sensors:

->  $n$  independent measurements

->  $n$  independent parameters estimable

-> at best separate activity from  $n$  brain regions

Sensors are not independent ->  $\sim 50$  degrees of freedom

Volume of source space:

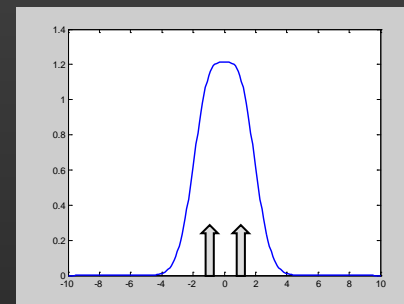
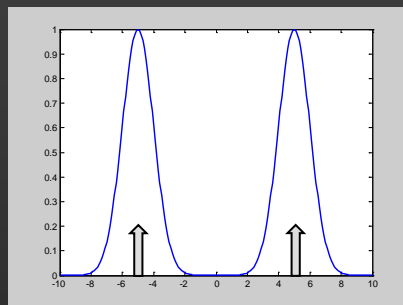
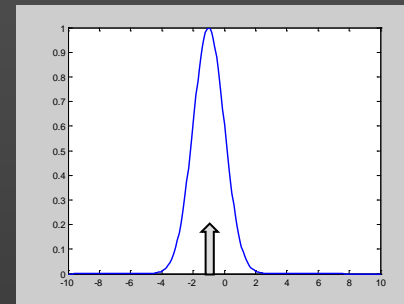
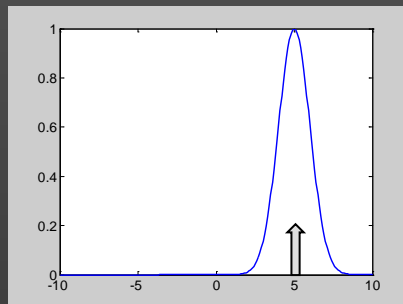
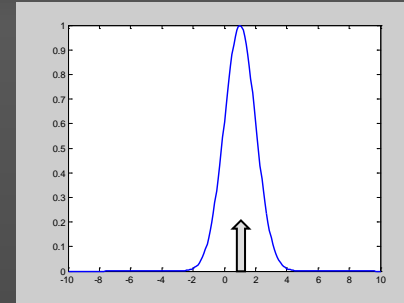
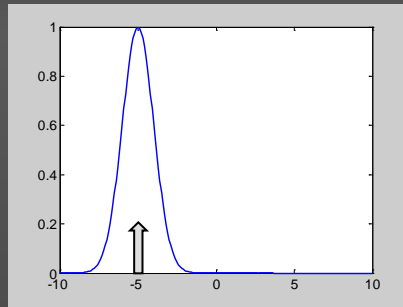
Sphere 8cm minus sphere 4 cm: volume  $\sim 1877 \text{ cm}^3$

“Resel”:  $38 \text{ cm}^3 \rightarrow \underline{3.4}^3 \text{ cm}^3$

The spatial resolution of the **measurement** is inherently limited!

# Linear Methods are Convenient Because Of...

## ...the Superposition Principle

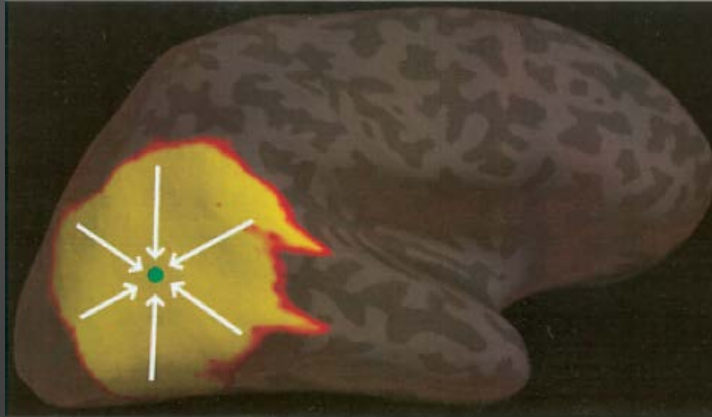


**If you know the behaviour for point sources,  
you can predict the behaviour for complex sources**

# Spatial Resolution:

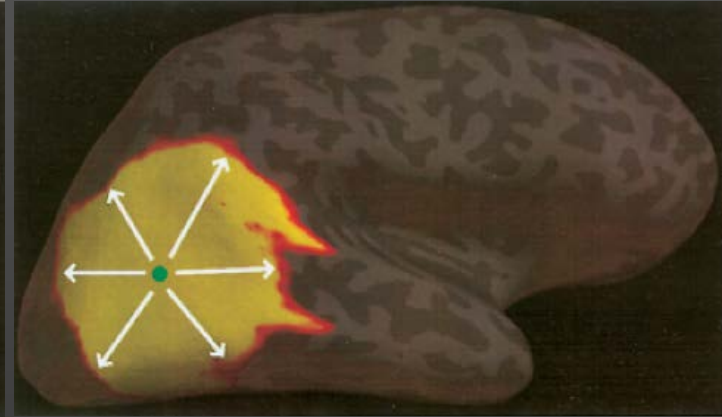
## Point-Spread and Cross-Talk/Leakage

### Cross-Talk Function (CTF)



*How other sources may affect the estimate for this source*

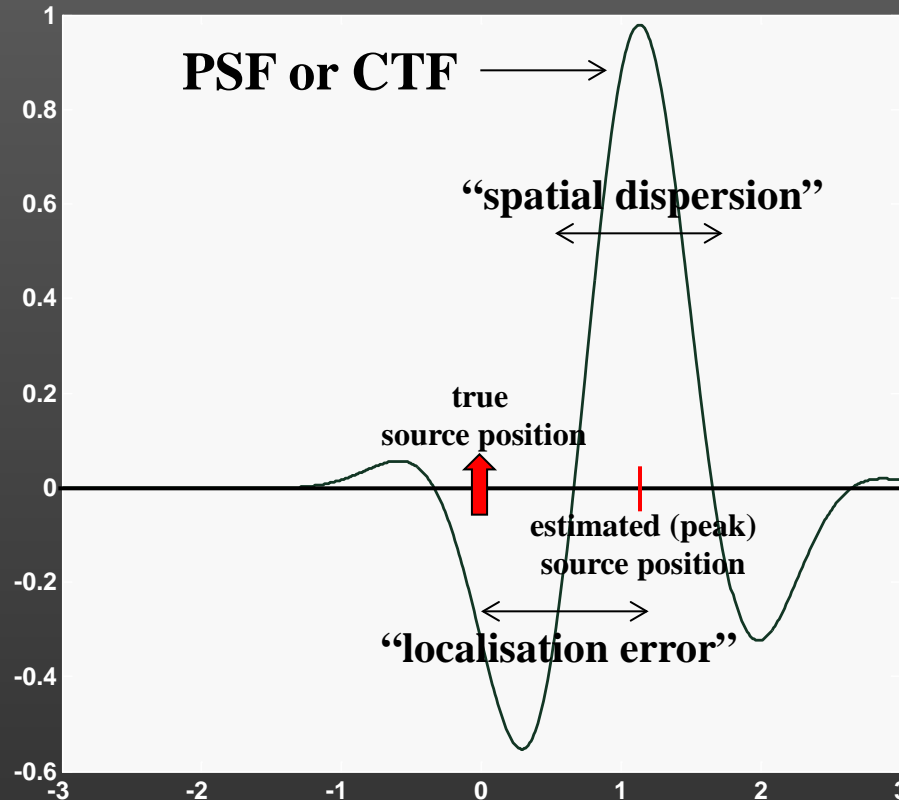
### Point-Spread Function (PSF)



*How this source affects estimates for other sources*

Liu et al., HBM 2002

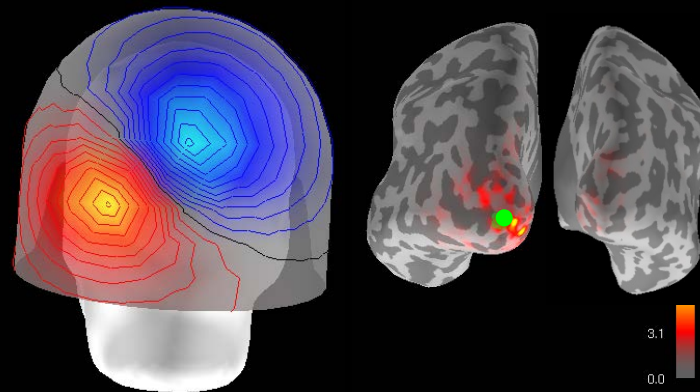
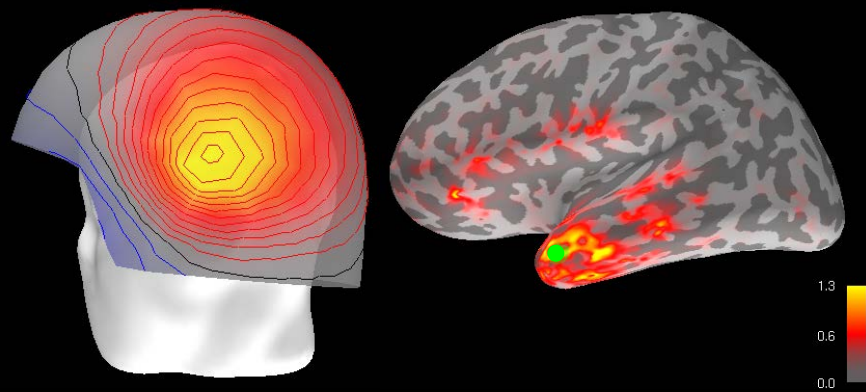
# Quantifying "Resolution"



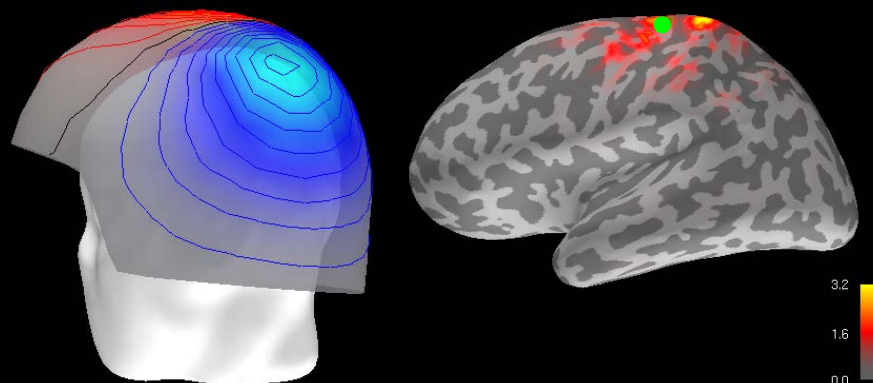
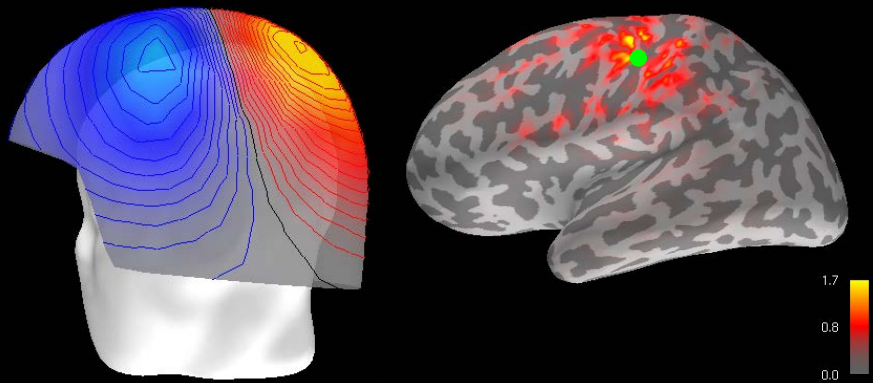
It's not just "peak localisation" that counts,  
but also spatial extent of the distribution ("resolution")

# PSFs and CTFs for Some ROIs

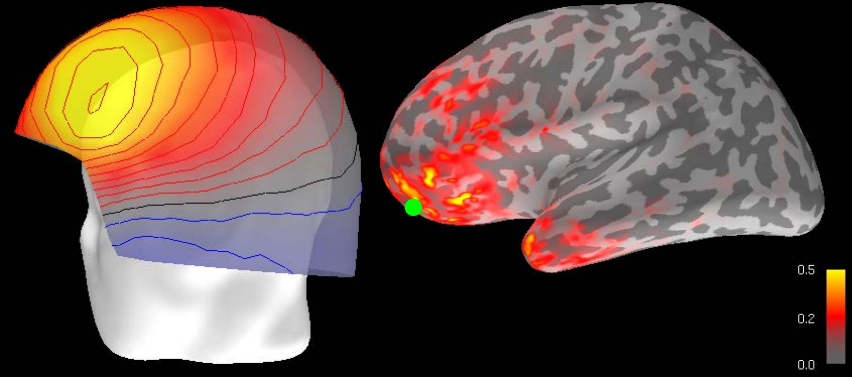
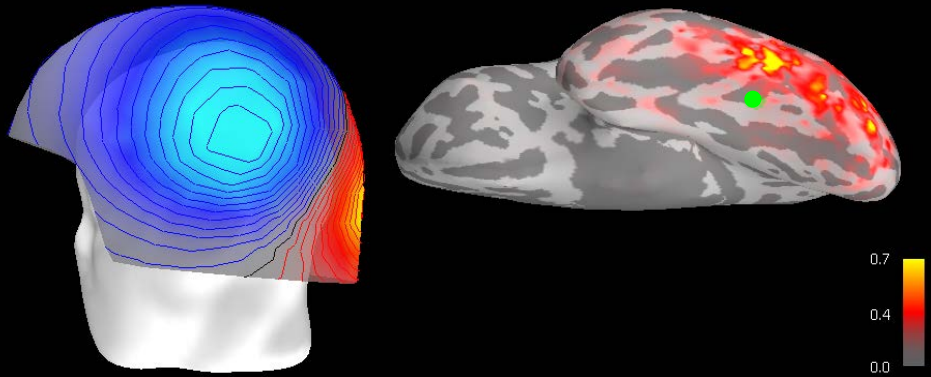
For MNE, PSFs and CTFs turn out to be the same



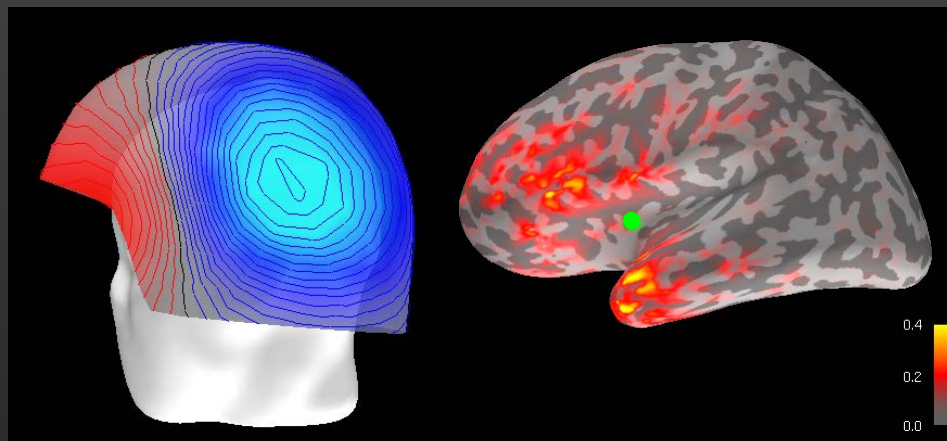
Good



# Localisation for Some ROIs



Less good



# Comparing Methods

**Different methods make different compromises.**

**There is no “best” method – best for what?**

**One should compare methods for the same purpose and under the same assumptions.**

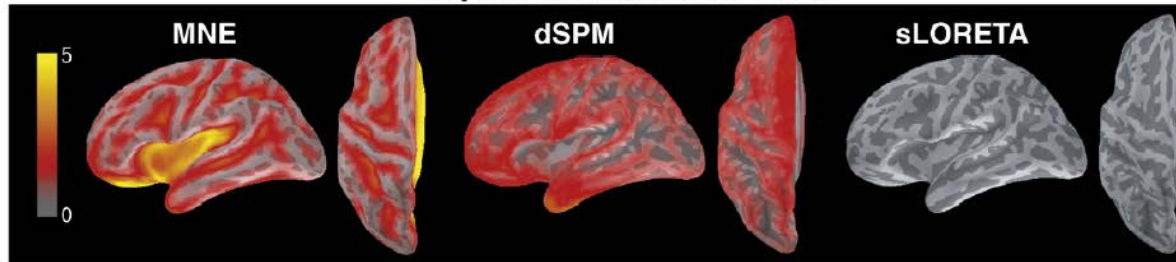
**Difficult to generalize results from one example or data set**

**=> Important to understand the principles**

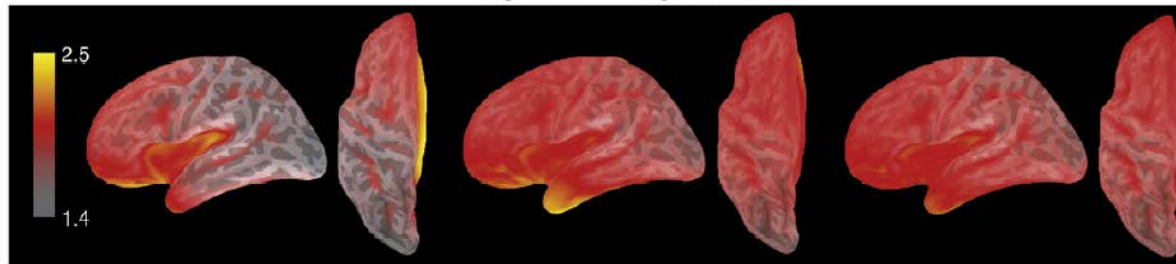


# Method Comparison

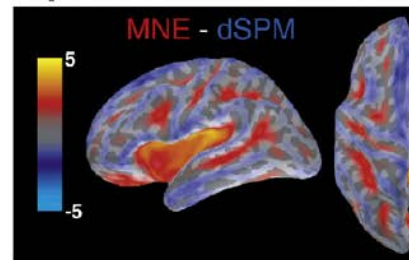
## Dipole Localization Error



## Spatial Dispersion



## Dipole Localization Error

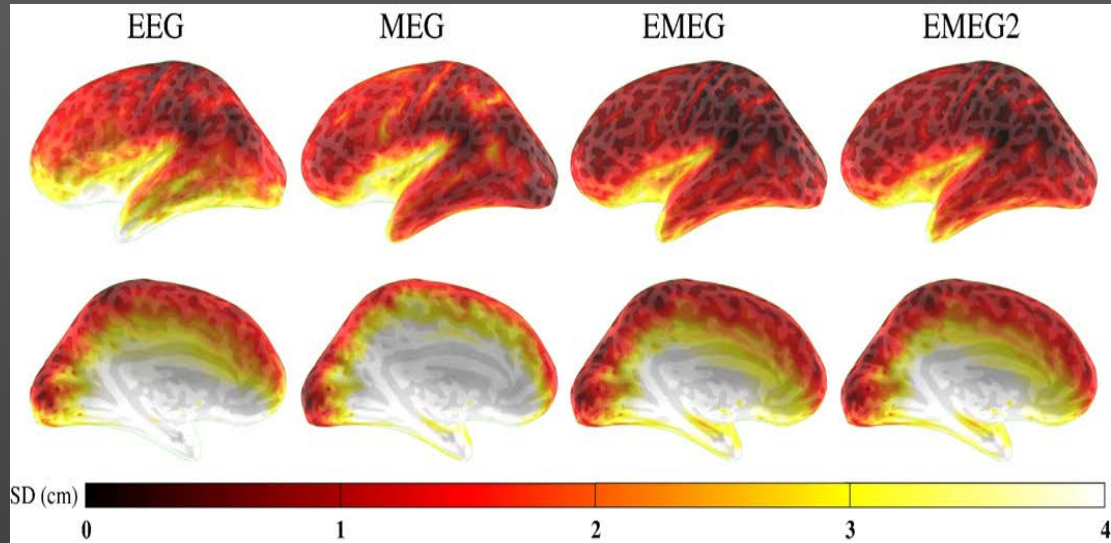


## Spatial Dispersion



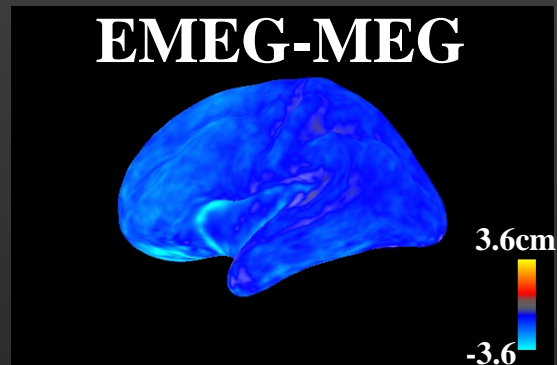
# Combining EEG and MEG Increases Resolution

## Spatial Extent



Molins et al., Neuroimage 2008

## EMEG-MEG



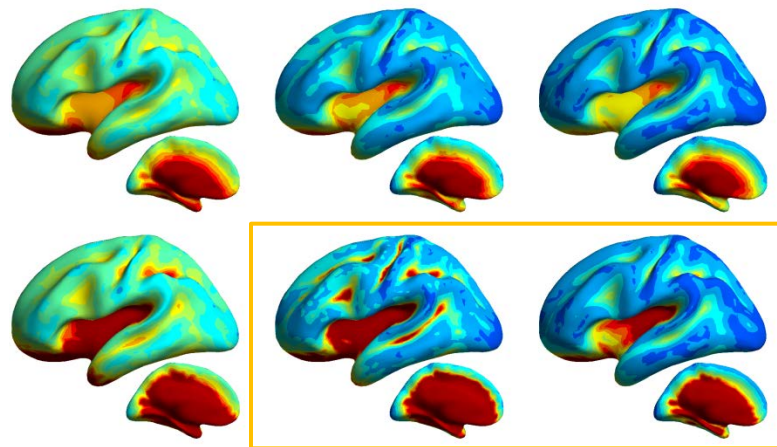
Stenroos&Hauk, in prep

# Combining EEG and MEG Improves Resolution

...especially in the presence of (correlated) noise

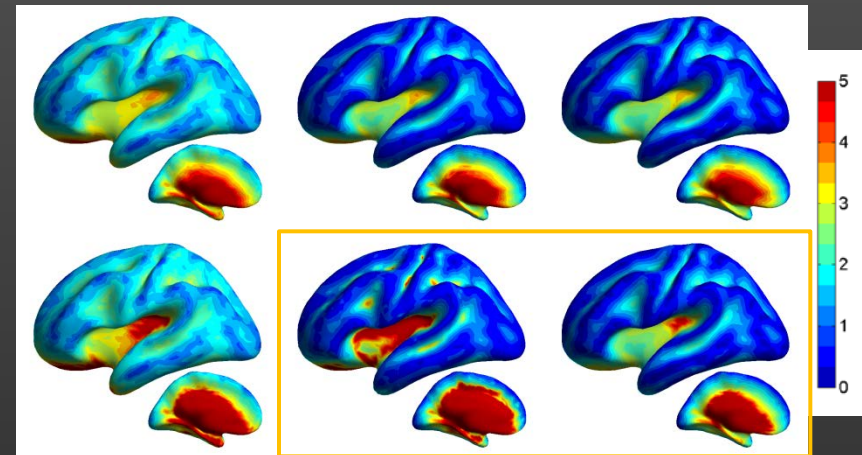
## Spatial Deviation (cm)

EEG      MEG      EMEG

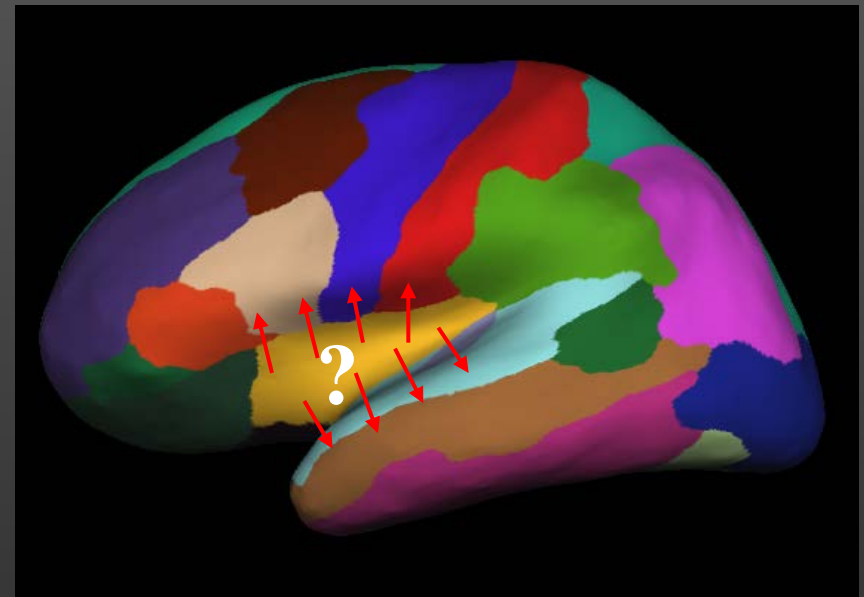
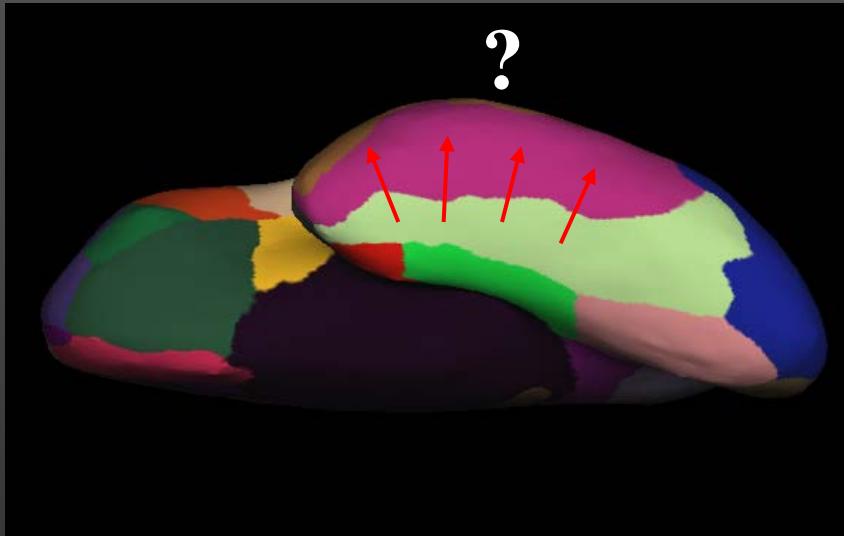


## Localisation Error (cm)

EEG      MEG      EMEG



# Localisation Bias Has Consequences for ROI analysis



Desikan-Killiany Atlas parcellation