Introduction to signal analysis in Matlab: GLM I

Ed Sohoglu

MRC Cognition and Brain Sciences Unit

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General Linear Model (GLM)

- Important statistical tool used in psychology and neuroscience for analysing behavioural data, EEG, MEG, fMRI etc.

- It is “general” because it can accommodate many different types of statistical tests all in the same framework (e.g. t-tests, regression, correlation, ANOVA etc.)

General Linear Model

$$Y = \beta 1X1 + \beta 2X2 + \ldots$$

- Y is the observed data (e.g. reaction time, BOLD activity)
- X is our ‘design matrix’ containing all our explanatory variables/regressors (e.g. condition, group or continuous covariates like age)
- β1 captures how much X1 explains the data in Y (and so on for β2 etc.)
- Usually we know Y and X so we try to solve for β

Solving for β: Numerical example

- Simple scenario: Y = β1X1
- If Y = 10 and X = 2, what is β?
- Use what we know from linear algebra
  - Y / X1 = β1X1 / X1
  - Or β1 = Y / X1
  - Or β1 = $YX1^{-1}$
- Try in Matlab: **Y = 10; X = 2;**
  B = Y * inv(X);

Solving for β: Extension to matrices

- Usually our data do not consist of a single number (Y = 10) but rather several numbers (Y = 10 5 6 2 1), one number for each trial or subject (N)

- Our design matrix X will also contain several numbers, arranged as a rectangular array (i.e. matrix) of N subjects (rows) by M variables/regressors (columns)

- This means we need the matrix formulation of the GLM:

$$\begin{pmatrix} Y_1 \\ Y_n \\ Y_N \end{pmatrix} = \begin{pmatrix} X_{11} \dots X_{1m} \dots X_{1M} \\ X_{n1} \dots X_{nm} \dots X_{nm} \\ X_{N1} \dots X_{Nm} \dots X_{NM} \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_m \\ \beta_M \end{pmatrix}$$

- Or more concisely: Y = XB

Solving for β: Matrix example

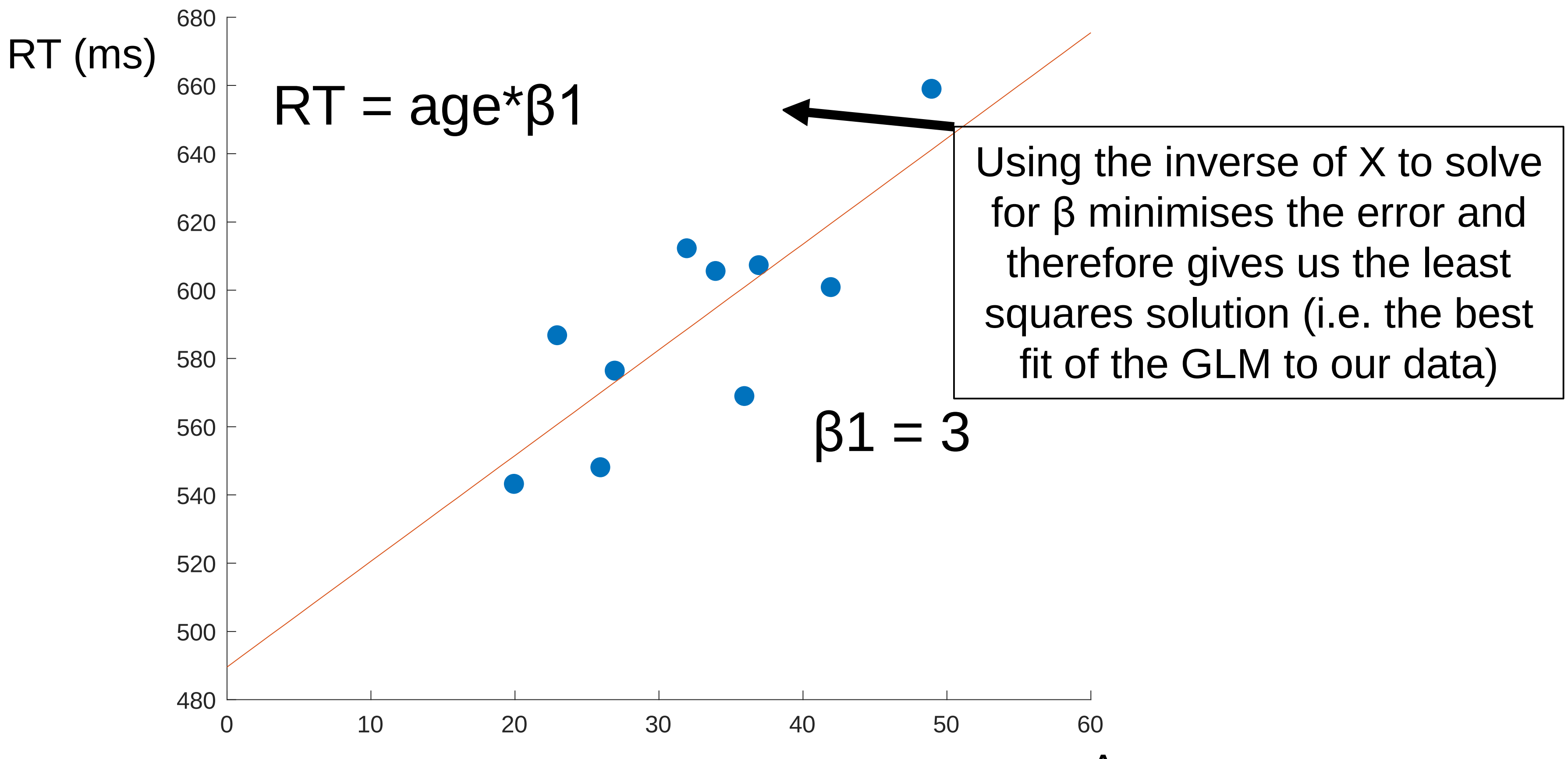
- As before, we are trying to solve for B in the equation Y = Xβ
- If Y = [-5 10] and X = [2 -1; 0 1], what is β?
- Solution same as before
  - $\beta = X^{-1}Y$
- Try in Matlab:
```
Y = [-5 10]'; X = [2 -1; 0 1];
B = inv(X) * Y;
```
- Note that matrix inversion is not the same as element-wise division!
- Compare in Matlab:
```
inv(X)
1./X
```
- So how do we know that the solution for B is correct? Because XB should equal Y.
- Check in Matlab:
```
X*B
```

Pseudoinverse

- The matrix inverse is only defined for square matrices. This is a problem if we have the typical situation of fewer explanatory variables than subjects/trials (i.e. the number of columns in X is less than the the number of rows)
- The matrix inverse is also only defined when none of our explanatory variables are linear combinations of other explanatory variables
- In these situations, the pseudoinverse can be used instead
- In Matlab: **pinv()**

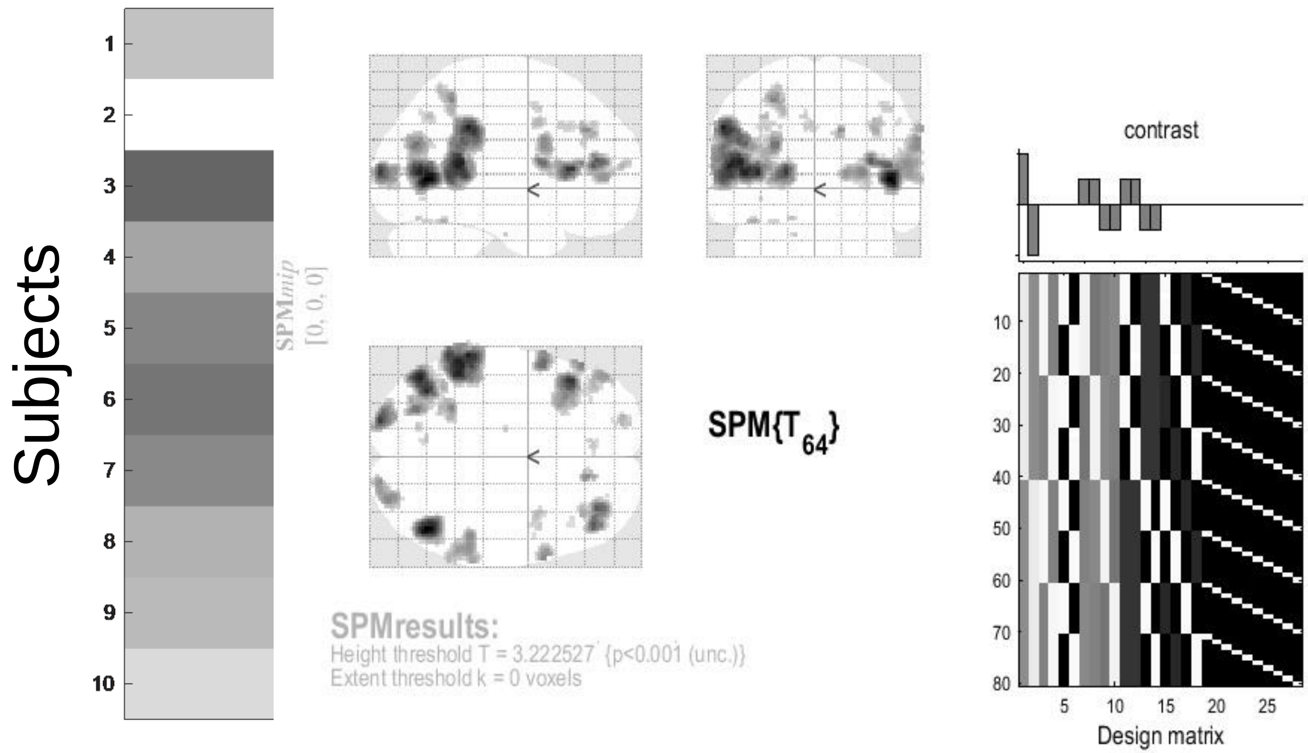
Graphical example

- Our data are mean reaction times (RTs) from 10 subjects.
- Use GLM to model RT as a function of subjects' age



Visualising the design matrix

- Commonly seen in neuroimaging software like SPM
- What does our design matrix for the age/reaction time experiment look like?



B1 (age)

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Useful references

- Handbook of Functional MRI Data Analysis (Poldrack, Mumford and Nichols 2011). Appendix A: “Review of the GLM”
- Cyril Pernet’s website http://www.sbirc.ed.ac.uk/cyril/glm/GLM_lectures.html
- Poline and Brett (2012) *NeuroImage* “The general linear model and fMRI: Does love last forever?”