Confidence Intervals for Cronbach's Coefficient Alpha Values

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Confidence Intervals for Comparing Cronbach's Coefficient Alpha Values ⋆

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Abstract

Coefficient alpha, which is widely used in empirical research, estimates the reliability of a test consisting of parallel items. In practice it is difficult to compare values of alpha across studies as it depends on the number of items used. In this paper we provide a simple solution, which amounts to computing the confidence intervals of an alpha, as these intervals automatically account for differences across the numbers of items. We also give appropriate statistics to test for significant differences of alpha values across studies.

Key words: Cronbach's alpha, test reliability, confidence intervals

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1 Introduction

Coefficient alpha, as introduced in Cronbach (1951), is a frequently used statistic in empirical research involving tests with various items. Under the assumption that the items are parallel, that is, the measurements have identical true scores and uncorrelated errors having equal variances, see Lord and Novick (1968), coefficient alpha can be interpreted as an estimator of the reliability of the test, see Cortina (1993). As a result of the well-known Spearman-Brown formula, discussed below, the reliability of the test depends on the true underlying correlation across items and also on the number of items. This complicates a comparison of estimated values of coefficient alpha across multiple studies or across replications.

In this paper we argue that a simple solution to this complication is to provide standard errors around estimated differences in values of coefficient alpha. As we will demonstrate below, these standard errors automatically capture differences across the number of items. A study related to ours is Iacobucci and Duhachek (2003), but we follow an alternative route towards formulating statistical tests. We will show that this route delivers more accurate inference in practice.

The outline of this paper is as follows. In section 2, we discuss some notation, the coefficient alpha, and its asymptotic distribution under linearly related measurements. In section 3, we show how confidence bounds can be derived for one single alpha. In section 4, we focus on comparing two alpha values corresponding to different studies. In section 5, we conclude with a few confidence intervals for typical practical situations. Proofs are relegated to an appendix.

As a courtesy to the reader, we also provide the relevant computer code in R.

2 Hypothesis and tests

In this section we first outline some notation around coefficient alpha. Next, we discuss asymptotic results obtained elsewhere in the literature.

2.1 Alpha

In classical test theory, see for example Lord and Novick (1968), Nunnally and Bernstein (1994), the objective is to approximate a latent - that is, unobserved - random variable, the true score T, by means of an observable random variable, the test score Y. It is generally agreed that the quality of the test score Y as a measurement device for the true score T is indicated by the reliability ρ^2_{TY}, where ρ_{TY} is the population correlation between T and Y. As the true score T is latent, the population correlation coefficient ρ_{TY} can not be estimated in the usual way, and alternative methods are required.

Typically, the test score Y is obtained by summing up a number [k, say] of item scores X_i, each measuring the T to some extent. The dependence between these item scores yields information with respect to the reliability ρ^2_{TY}.

Cronbach's alpha, introduced in Cronbach (1951), is defined as

$$\alpha = \frac{k}{k-1}\left(1 - \frac{\mathrm{tr}\boldsymbol{\Phi}}{\iota^T\boldsymbol{\Phi}\iota}\right), \tag{1}$$

where $\boldsymbol{\Phi}$ is the covariance matrix of the item scores, and ι is a p-vector of ones. When the test is administered to a random sample of subjects, Cronbach's alpha is estimated by

$$\hat{\alpha} = \frac{k}{k-1}\left(1 - \frac{\mathrm{tr}\mathbf{S}}{\iota^T\mathbf{S}\iota}\right), \tag{2}$$

where $\mathbf{S}$ is the unbiased estimator of the item score covariance matrix $\mathbf{\Phi}$.

The estimator $\hat{\alpha}$ is often encountered in applied statistical work which involves latent random variables, for example, the evaluation of the quality of a questionnaire as a measuring device for some latent trait. However, seldom the distinction is made between the reliability coefficient α defined by (1) and its estimator $\hat{\alpha}$ defined by (2). In this paper we therefore examine what $\hat{\alpha}$ says about α, and hence about ρ. We shall see that this turns out to be useful when comparing various values of $\hat{\alpha}$ obtained in different studies.

In this paper we focus on the special situation where the items are parallel, that is, the items have identical true scores and uncorrelated measurement errors. In this case, the item score covariance matrix $\mathbf{\Phi}$ has compound symmetry, that is,

$$\mathbf{\Phi} = \phi^2 \begin{pmatrix} 1 & \rho & \cdots & \rho \\ \rho & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \rho \\ \rho & \cdots & \rho & 1 \end{pmatrix}. \tag{3}$$

Observe that ρ is the population correlation between two different item scores X_i and X_j. Throughout this paper we shall assume that ρ is non-negative, which seems useful given the very purpose of the items of a test.

When the items are parallel, coefficient alpha coincides with the general Spearman-Brown formula, that is,

$$\alpha = \frac{k\rho}{1+(k-1)\rho}, \tag{4}$$

see Lord and Novick (1968, page 90) and Winer (1991). As noted before, the Spearman-Brown formula implies that the reliability of the test depends on the true underlying correlation across items, and on the number of items. Iacobucci and Duhachek (2003) show that α approximately satisfies

$$\alpha = -0.022 + 0.023k + 2.055\rho - 1.207\left(\rho\right)^2,$$

but, of course, (4) gives an exact description of the dependence of α on k and ρ.

2.2 Asymptotic results

Recently, Iacobucci and Duhachek (2003) derived an asymptotic confidence interval for α by using that, as n tends to infinity, $\sqrt{n}\,(\hat{\alpha}-\alpha)$ tends in distribution to a normal random variable with expectation 0 and variance $(k/(k-1))^2\omega$, see van Zyl et al. (2000, Equation (21)). Here n denotes the number of subjects to which the test is administered, and ω is given by

$$\omega = \frac{2}{(\iota^T\mathbf{\Phi}\iota)^3}\left\{(\iota^T\mathbf{\Phi}\iota)\left(\mathrm{tr}\mathbf{\Phi}^2+\mathrm{tr}^2\mathbf{\Phi}\right)-2(\mathrm{tr}\mathbf{\Phi})(\iota^T\mathbf{\Phi}^2\iota)^2\right\},$$

see van Zyl et al. (2000, Equation (20)). If the items are parallel, then ω simplifies to

$$\omega = \frac{2(k-1)}{k}\left(\frac{1-\rho}{1+(k-1)\rho)}\right)^2 = \frac{2(k-1)}{k}\left(1-\alpha\right)^2,$$

see van Zyl et al. (2000, Equation (22)). Hence, if the items are parallel, then, as n tends to infinity, $\sqrt{n}\,(\hat{\alpha} - \alpha)$ tends in distribution to a normal random variable with expectation zero and variance $(2k/(k-1))(1-\alpha)^2)$, see van Zyl et al. (2000, Equation (22)).

Now, observe that the asymptotic variance of $\sqrt{n}\,(\hat{\alpha} - \alpha)$ depends on α. This means that $\sqrt{n}\,(\hat{\alpha} - \alpha)$ does not have a stable variance. Generally, this may be viewed as a shortcoming for statistics from which confidence intervals are to be derived. Indeed, and not mentioned explicitly, the asymptotic variance $Q = (2k/(k-1))(1-\alpha)^2$ is estimated by $\hat{Q} = (2k/(k-1))(1-\hat{\alpha})^2$ in Iacobucci and Duhachek (2003). It is well-known that using the estimated asymptotic variance may well yield anti-conservative confidence intervals. Anti-conservative confidence intervals have a nominal coverage less than the required coverage γ, and hence may falsely give the impression that α is estimated with sufficient accuracy.

However, we may obtain a "stable" confidence interval by using other results in van Zyl et al. (2000), and this is what we advocate in this paper.

If the item scores $X_1, X_2, \ldots, X_k$ are parallel, and have a joint multivariate normal distribution, then the random variable

$$\frac{1-\hat{\alpha}}{1-\alpha} = \frac{1-\hat{\alpha}}{1-\frac{1-\rho}{1+(k-1)\rho}} \tag{5}$$

has an F distribution with $n(k-1)$ degrees of freedom in the nominator, and n degrees of freedom in the denominator, see Kristof (1963), Feldt (1965) and van Zyl et al. (2000, Equation (7)). Hence, it is a pivotal quantity for α, see Mood et al. (1974, Section 2.3). The equality in (5) follows from the general Spearman-Brown formula (4). Since $\rho \geq 0$ implies $(1-\rho)/(1+(k-1)\rho) \geq 0$, we have $1 - \hat{\alpha} \geq 0$ due to the assumed non-negativeness of ρ.

The F distribution with $n(k-1)$ and n degrees of freedom has expectation $n/(n-2)$ and variance

$$\frac{2n^2(nk-2)}{n(k-1)(n-2)^2(n-4)} = \frac{2}{n-4}\left(\frac{n}{n-2}\right)^2 \frac{k-2/n}{k-1},$$

see Mood et al. (1974, p. 248), and tends to a normal distribution with expectation 1 and asymptotic variance $2k/n(k-1)$. Hence, the fact that (5) has an F-distribution with $n(k-1)$ and n degrees of freedom implies that $\sqrt{n}\,(\hat{\alpha} - \alpha)$ tends in distribution to a normal random variable with expectation zero and variance $(2k/(k-1))(1-\alpha)^2)$.

Moreover, it follows by applying the z-transformation of Fisher (1967, p. 219) to the random variable (5) that

$$\sqrt{\frac{2n(k-1)}{k}}\left(\tfrac{1}{2}\ln(1-\hat{\alpha}) - \tfrac{1}{2}\ln(1-\alpha)\right) \tag{6}$$

tends to a standard normal random variable Z in distribution, as n tends to infinity, see van Zyl et al. (2000, Equation (14)). As its asympotic distribution does not depend on α, (6) is an asymptotic pivotal quantity for α. The expression in (6) suggests that for larger n, the confidence intervals become smaller. However, for fixed values of the n and $\hat{\alpha}$, the length of the interval tends to a non-zero limit as k becomes large. In section 5 we will examine to what extent this holds true.

3 Confidence intervals for Cronbach's alpha

Let $0 < \gamma < 1$ be a confidence level, where a common choice for γ is 95%. Let z_L and z_R be values such that

$$P(Z \leq z_L) = P(Z \leq z_R) = \tfrac{1}{2}(1-\gamma), \tag{7}$$

with Z denoting a standard normal random variable. The symmetry of the standard normal distribution around zero implies that $z_L = -z_R$. For $\gamma = 0.95$, we have $z_L = -1.96$ and $z_R = 1.96$.

We start out by presenting the asymptotic $100\gamma\%$-confidence interval for α described in Iacobucci and Duhachek (2003), to which we shall refer as the ID interval. This interval is based on the asymptotic normality of $\sqrt{n}\,(\hat{\alpha} - \alpha)$. Define

$$\alpha_L^{(\mathrm{ID})} = \hat{\alpha} - z_R(1-\hat{\alpha})\sqrt{\frac{2k}{n(k-1)}}$$

$$\alpha_R^{(\mathrm{ID})} = \hat{\alpha} - z_L(1-\hat{\alpha})\sqrt{\frac{2k}{n(k-1)}}$$

Then,

$$\lim_{n\to\infty} P\left(\alpha_L^{(\mathrm{ID})} \leq \alpha \leq \alpha_R^{(\mathrm{ID})}\right) = \gamma, \tag{8}$$

that is, the interval with endpoints $\alpha_L^{(\mathrm{ID})}$ and $\alpha_R^{(\mathrm{ID})}$ is an asymptotic $100\gamma\%$-confidence interval for α. Note that this confidence interval is symmetric around $\hat{\alpha}$.

Alternatively, which is what we prefer for earlier mentioned reasons, we may base an asymptotic confidence interval on the asymptotic normality of the random variable (6), which has stable asymptotic variance. Define

$$\alpha_L^{(z)} = 1 - (1-\hat{\alpha}) \exp\left\{-z_L \sqrt{\frac{2k}{n(k-1)}}\right\},$$

$$\alpha_R^{(z)} = 1 - (1-\hat{\alpha}) \exp\left\{-z_R \sqrt{\frac{2k}{n(k-1)}}\right\}.$$

Then,

$$\lim_{n\to\infty} P\left(\alpha_L^{(z)} \le \alpha \le \alpha_R^{(z)}\right) = \gamma, \tag{9}$$

see Appendix A for a proof. That is, the interval with endpoints $\alpha_L^{(z)}$ and $\alpha_R^{(z)}$ is an asymptotic γ-confidence interval for α. Note that this confidence interval is not symmetric around $\hat{\alpha}$, and that it is skewed to the left.

If in addition the item scores $X_1, X_2, \ldots, X_k$ have a joint multivariate normal distribution, then it is even possible to construct an exact γ-confidence interval for α. The exact confidence interval is based on the observation that

$$F = \frac{1-\hat{\alpha}}{1-\alpha} \tag{10}$$

is a pivotal quantity for α, which follows an F-distribution with $n(k-1)$ degrees of freedom in the nominator, and n degrees of freedom in the denominator. Let F_L and F_R be values such that

$$P\left(F \le F_L\right) = P\left(F \le F_R\right) = \tfrac{1}{2}(1-\gamma), \tag{11}$$

and define

$$\alpha_L^{(F)} = 1 - \frac{1-R}{F_L}, \quad \alpha_R^{(F)} = 1 - \frac{1-R}{F_R}.$$

Then,

$$P\left(\alpha_L^{(F)} \leq \alpha \leq \alpha_R^{(F)}\right) = \gamma, \tag{12}$$

see Appendix A for a proof. That is, the interval with endpoints $\alpha_L^{(F)}$ and $\alpha_R^{(F)}$ is an exact γ-confidence interval for α. In appendix B we provide the relevant computer code in R, as a courtesy to the reader.

4 Reliability comparisons

In this section we present confidence intervals which allow to assess whether two independently estimated values of α are significantly different.

The setup is as follows. We consider two independent studies. The first study reports an estimated Cronbach's alpha $\hat{\alpha}_1$ for a scale with k_1 parallel items which has been administered to a random sample of n_1 subjects, while the second study reports an estimated Cronbach's alpha $\hat{\alpha}_2$ for a scale with k_2 parallel items which has been administered to a random sample of n_2 subjects.

As $\sqrt{n_i}\,(\hat{\alpha}_i - \alpha_i)$ tends in distribution to a normal random variable with expectation zero and variance $(2k_1/(k_i-1))(1-\alpha_i)^2$, we obtain that the distribution of $(\hat{\alpha}_1 - \hat{\alpha}_2) - (\alpha_1 - \alpha_2)$ approximately follows a normal distribution with expectation zero and variance

$$\frac{2k_1}{n_1(k_1-1)}(1-\alpha_1)^2 + \frac{2k_2}{n_2(k_2-1)}(1-\alpha_2)^2.$$

Note that this variance is not stable, as it depends on both α_1 and α_2. Hence, we are now forced to use the estimated variance

$$\frac{2k_1}{n_1(k_1-1)}(1-\hat{\alpha}_1)^2+\frac{2k_2}{n_2(k_2-1)}(1-\hat{\alpha}_2)^2$$

instead. That is, we extend the approach in Iacobucci and Duhachek (2003). Let $0<\gamma<1$ be a confidence level, and let z_L and z_R be as before. Define

$$\delta_L=(\hat{\alpha}_1-\hat{\alpha}_2)+z_L\sqrt{\frac{2k_1}{n_1(k_1-1)}(1-\hat{\alpha}_1)^2+\frac{2k_2}{n_2(k_2-1)}(1-\hat{\alpha}_2)^2},$$

$$\delta_R=(\hat{\alpha}_1-\hat{\alpha}_2)+z_R\sqrt{\frac{2k_1}{n_1(k_1-1)}(1-\hat{\alpha}_1)^2+\frac{2k_2}{n_2(k_2-1)}(1-\hat{\alpha}_2)^2}.$$

Then,

$$\lim_{\min(n_1,n_2)\to\infty} P\left(\delta_L\le\alpha_1-\alpha_2\le\delta_R\right)=\gamma, \tag{13}$$

That is, the interval with endpoints δ_L and δ_R is an asymptotic $100\gamma\%$-confidence interval for $\alpha_1-\alpha_2$. Note that this confidence interval is symmetric around $\hat{\alpha}_1-\hat{\alpha}_2$.

The fact that the random variable $\ln\left(\frac{1-\hat{\alpha}_i}{1-\alpha_i}\right)$ has an F-distribution with $n_i(k_i-1)$ and n_i degrees of freedom allows us to derive an exact confidence interval for $\ln\left(\frac{1-\alpha_1}{1-\alpha_2}\right)$. In this respect, observe that the random variable

$$G=\ln\left(\frac{1-\hat{\alpha}_1}{1-\hat{\alpha}_2}\right)-\ln\left(\frac{1-\alpha_1}{1-\alpha_2}\right)=\ln\left(\frac{1-\hat{\alpha}_1}{1-\alpha_1}\right)-\ln\left(\frac{1-\hat{\alpha}_2}{1-\alpha_2}\right) \tag{14}$$

is the difference between the logarithms of two independent F-distributed random variables. with $n_i(k_i-1)$ and n_i degrees of freedom.

Relevant percentage points for the distribution of G are given in Tables 1, 2 and 3. Let G_L and G_R be lower and upper percentage points such that

$$P(G \leq G_L) = P(G \geq G_R) = \tfrac{1}{2}(1-\gamma), \tag{15}$$

and define

$$\tau_L = \ln\left(\frac{1-\hat{\alpha}_1}{1-\hat{\alpha}_2}\right) - G_R,$$

$$\tau_R = \ln\left(\frac{1-\hat{\alpha}_1}{1-\hat{\alpha}_2}\right) - G_L$$

Then,

$$\lim_{n\to\infty} P\left(G_L < \ln\left(\frac{1-\alpha_1}{1-\alpha_2}\right) < G_L\right) = \gamma, \tag{16}$$

That is, the interval with endpoints τ_L and τ_R is an exact $100\gamma\%$-confidence interval for $\ln\left(\frac{1-\alpha_1}{1-\alpha_2}\right)$. Observe that

$$\ln\left(\frac{1-\alpha_1}{1-\alpha_2}\right) \approx \alpha_1 - \alpha_2.$$

In particular, we have

$$\ln\left(\frac{1-\alpha_1}{1-\alpha_2}\right) = 0 \Leftrightarrow \alpha_1 - \alpha_2 = 0 \Leftrightarrow \alpha_1 = \alpha_2.$$

Thus, we may interpret the absence of the value zero in the exact $100\gamma\%$ confidence interval for $\ln\left(\frac{1-\alpha_1}{1-\alpha_2}\right)$ as a rejection of the null hypothesis $H_0 : \alpha_1 = \alpha_2$ using a two-sided test with significance level $1-\gamma$.

As the random variable $\ln\left(\frac{1-\hat{\alpha}_i}{1-\alpha_i}\right)$ tends in distribution to a normal random variable with expectation zero and variance $2k_i/(k_i-1)$, we obtain that G tends in distribution to a normal random variable with expectation zero and variance

$$\frac{2k_1}{n_1(k_1-1)}+\frac{2k_2}{n_2(k_2-1)},$$

and hence

$$G_L \approx z_L\sqrt{\frac{2k_1}{n_1(k_1-1)}+\frac{2k_2}{n_2(k_2-1)}},$$

$$G_R \approx z_R\sqrt{\frac{2k_1}{n_1(k_1-1)}+\frac{2k_2}{n_2(k_2-1)}}.$$

In other words, the interval with endpoints

$$\ln\left(\frac{1-\hat{\alpha}_1}{1-\hat{\alpha}_2}\right)+z_L\sqrt{\frac{2k_1}{n_1(k_1-1)}+\frac{2k_2}{n_2(k_2-1)}},$$

$$\ln\left(\frac{1-\hat{\alpha}_1}{1-\hat{\alpha}_2}\right)+z_R\sqrt{\frac{2k_1}{n_1(k_1-1)}+\frac{2k_2}{n_2(k_2-1)}}$$

is an asymptotic $100\gamma\%$-confidence interval for $\ln\left(\frac{1-\alpha_1}{1-\alpha_2}\right)$.

5 Illustrations

In Tables 4 to 7 we give some 95% confidence intervals for a few values of $\hat{\alpha}$, n and k. As predicted, these intervals get narrower when n increases. So, the more subjects involved in a study, the more likely it becomes that one can significantly distinguish estimates alpha values across empirical studies. Next, with increasing values of k, these intervals also narrow down, but as $\frac{k-1}{k}$ approaches 1 for large k, this narrowing becomes less relevant.

The simulated nominal coverages of the intervals are explored in Tables 4 to 7. The ID interval is anti-conservative, especially for $k = 2$ and $n = 50$.

The proposed asymptotic interval is less anti-conservative. For the proposed exact interval, the simulated nominal coverage is approximately equal to the confidence level $\gamma = 0.95$. This is confirmed by simulated nominal coverage levels reported in Tables 8 to 11. For large values of k and n, the differences across methods get smaller.

Figures 1 to 3 shed some additional interesting light on the behavior of the confidence intervals for α. In these figures we simulated estimated values of α for a given "true" value of α, say α_0, and recorded the nominal coverage of the 95% confidence intervals for various other values of α. Ideally, the nominal coverage of a 95% confidence interval for α should be exactly 0.9500 for $\alpha = \alpha_0$ and should be less than 0.95 (but prefferably as small as possible) for $\alpha \neq \alpha_0$. The plots confirm the anti-conservatism of the ID interval, but also reveal that its nominal coverage exceeds 0.9500 for values of α slightly larger than α_0. In this sense, the ID interval is biased. Our proposed intervals show better behavior. With respect to the exact interval, the asymptotic proposed interval is less sensitive than values of α larger than α_0, and more sensitive for values of α less than α_0. This difference in sensitivity is hardly noticeable for $k = 2$, but becomes larger as k increases. The difference in nominal coverage between the ID interval and the proposed exact interval becomes smaller as k increases. Figures constructed for other "true" values α_0 show similar behavior of the three intervals, but are not reproduced in this paper to save space.

In Tables 12, 13 and 14 we list 95% confidence bounds for $\delta = \alpha_1 - \alpha_2$ and $\tau = \ln\left(\frac{1-\alpha_2}{1-\alpha_1}\right)$, given that the estimated Cronbach's coefficients $\hat{\alpha}_1$ and $\hat{\alpha}_2$ take the values 0.7 and 0.8. The intervals get narrower when the sample size n_1 and n_2 increase. So, the more subjects involved in a study, the more likely it becomes that one can significantly distinguish estimated alpha values across empirical studies. The intervals also narrow down with increasing values of k_1 and k_2, but this narrowing becomes less relevant for large values of k_1 and k_2.

Figure 4 is the two-sample counterpart to Figures 1 to 3, and shows contour plots of simulated nominal coverage of 95% asymptotic confidence bounds for reliability comparisons. We simulated estimated values of α_1 and α_2 for given "true" values of α_1 and α_2, say $\alpha_{1,0}$ and $\alpha_{2,0}$, and recorded the nominal coverage of the 95% confidence intervals for various other values of α_1 and α_2. The 0.96 contour line only emerges in the plots for the asymptotic confidence interval for $\delta = \alpha_1 - \alpha_2$, confirming that this interval is indeed conservative. However, the interval does not seem to suffer from bias, although it is related to the ID interval. As their contour lines are narrower, the asymptotic and exact confidence interval for $\tau = \log(1 - \alpha_2) - \log(1 - \alpha_1)$ are more sensitive than the asymptotic confidence interval for δ for larger values of α_1 and α_2. Figures constructed for other "true" values $\alpha_{1,0}$ and $\alpha_{2,0}$ show similar behavior of the three intervals, but are not reproduced in this paper.

Finally, a remark for practical use. The empirical relevance of these confidence intervals for comparing practical studies can be illustrated by the following example. Suppose study A has relied on 100 subjects, used a test with 4 items and reports an estimated alpha of 0.7. Study B, addressing the same topic, also used 100 (but different) subjects, but used a test with 6 items, reports an alpha of 0.8. Both confidence intervals for $\tau = \ln\left(\frac{1-\alpha_2}{1-\alpha_1}\right)$ in Table 13 do not contain the value zero, indicating that there is a significant difference between the two coefficients. In contrast, the asymptotic confidence for $\delta = \alpha_1 - \alpha_2$ does contain the value zero, which means that no significant difference between α_1

and α_2 was found.

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Appendix A: Proofs

Proof of Equation (9) The convergence in distribution of the random variable (6) to Z implies

$$\lim_{n\to\infty} P\left(z_L \le Z(\alpha) \le z_R\right) = \gamma, \tag{17}$$

where

$$Z(\alpha) = \sqrt{\frac{n(k-1)}{2k}}\left(\ln(1-\hat{\alpha}) - \ln(1-\alpha)\right)$$

is a [random] increasing function of α. The inverse $Z^{-1}(z)$ of this function is given by

$$Z^{-1}(z) = 1 - (1-\hat{\alpha})\exp\left\{-z\sqrt{2\rho/n(k-1)}\right\}.$$

Since $Z^{-1}(z)$ is increasing, (17) yields

$$\lim_{n\to\infty} P\left(Z^{-1}\left(z_L\right) \le Z^{-1}\left(Z(\alpha)\right) \le Z^{-1}\left(z_R\right)\right) = \gamma.$$

Observing that $Z^{-1}\left(z_L\right) = \alpha_L^{(z)}$, $Z^{-1}\left(Z(\alpha)\right) = \alpha$ and $Z^{-1}\left(z_R\right) = \alpha_R^{(z)}$ concludes the proof of Equation (9). □

Proof of Equation (12) We start by observing that (4) implies

$$\frac{1-\rho}{1+(k-1)\rho} = 1-\alpha$$

As a consequence of Equation (5), we may write

$$\lim_{n\to\infty} P\left(F_L \le F(\alpha) \le F_R\right) = \gamma, \tag{18}$$

where

$$F(\alpha) = \frac{1-R}{1-\alpha}$$

is a [random] increasing function of α. The inverse $F^{-1}(f)$ of this function is given by

$$F^{-1}(f) = 1 - \frac{1-R}{f}.$$

Since $F^{-1}(f)$ is increasing function of f, (18) yields

$$P\left(F^{-1}\left(F_L\right) \leq F^{-1}\left(F(\alpha)\right) \leq F^{-1}\left(F_R\right)\right) = \gamma.$$

By observing that $F^{-1}\left(F_L\right) = \alpha_L^{(F)}$, $F^{-1}\left(F(\alpha)\right) = \alpha$ and $F^{-1}\left(F_R\right) = \alpha_R^{(F)}$, Equation (12) readily follows. $\square$

Appendix B: R code

R, see Ihaka and Gentleman (1996) and see also Cribari-Neto and Zarkos (1999), is a language and environment for statistical computing and graphics. It is a GNU project which is similar to the S language and environment which was developed at Bell Laboratories (formerly AT&T, now Lucent Technologies) by John Chambers and colleagues. R can be considered as a different implementation of S. There are some important differences, but much code written for S runs unaltered under R. A Web based interface to R is available.

The function `alpha.IDci` computes the lower bound $\alpha_L^{(\mathrm{ID})}$ and upper bound $\alpha_R^{(\mathrm{ID})}$ of the ID confidence interval for Cronbach's coefficient α, see (9). The arguments are $\hat{\alpha}$, k, n and γ.

```
alpha.IDci <- function(R,nitem,nsample,clevel) {
 pcrit <- 0.5*(1-clevel)
 zcritl <- qnorm(pcrit,lower.tail=TRUE)
 zcritr <- qnorm(pcrit,lower.tail=FALSE)
 w <- sqrt(2*nitem/(nsample*(nitem-1)))
 IDcil <- R-zcritr*(1-R)*w
 IDcir <- R-zcritl*(1-R)*w
 list(lower=IDcil,upper=IDcir)
}
```

The function `alpha.aci` computes the lower bound $\alpha_L^{(z)}$ and upper bound $\alpha_R^{(z)}$ of the proposed asymptotic confidence interval for Cronbach's coefficient α, see (9). The arguments are $\hat{\alpha}$, k, n and γ.

```
alpha.aci <- function(R,nitem,nsample,clevel) {
```

```
 pcrit <- 0.5*(1-clevel)
 zcritl <- qnorm(pcrit,lower.tail=TRUE)
 zcritr <- qnorm(pcrit,lower.tail=FALSE)
 w <- sqrt(2*nitem/(nsample*(nitem-1)))
 acil <- 1-(1-R)*exp(zcritr*w)
 acir <- 1-(1-R)*exp(zcritl*w)
 list(lower=acil,upper=acir)
}
```

The function `alpha.xci` computes the lower bound $\alpha_L^{(F)}$ and upper bound $\alpha_R^{(F)}$ of the proposed exact confidence interval for Cronbach's coefficient α, see (12). The arguments are $\hat{\alpha}$, k, n and γ.

```
alpha.xci <- function(R,nitem,nsample,clevel) {
 pcrit <- 0.5*(1-clevel)
 df1 <- nsample*(nitem-1)
 df2 <- nsample
 fcritl <- qf(pcrit,df1,df2,lower.tail=TRUE)
 fcritr <- qf(pcrit,df1,df2,lower.tail=FALSE)
 xcil <- 1-((1-R)/fcritl)
 xcir <- 1-((1-R)/fcritr)
 list(lower=xcil,upper=xcir)
}
```

The function `delta.aci` computes the lower bound δ_L and upper bound δ_R of the asymptotic confidence interval for the difference $\delta = \alpha_1 - \alpha_2$ of two Cronbach's coefficients, see (13). The arguments are $\hat{\alpha}_1$, k_1, n_1, $\hat{\alpha}_2$, k_2, n_2 and γ.

```
delta.aci <- function(R1,nitem1,nsample1,R2,nitem2,nsample2,clevel) {
 pcrit <- 0.5*(1-clevel)
 zcritl <- qnorm(pcrit,lower.tail=TRUE)
 zcritr <- qnorm(pcrit,lower.tail=FALSE)
 ww1 <- 2*nitem1/(nsample1*(nitem1-1))
 ww2 <- 2*nitem2/(nsample2*(nitem2-1))
 w <- sqrt(ww1*(1-R1)*(1-R1)+ww2*(1-R2)*(1-R2))
 D <- R1-R2
 acil <- D-zcritr*w
 acir <- D-zcritl*w
 list(lower=acil,upper=acir)
}
```

The function `tau.aci` computes the lower bound τ_L and upper bound τ_R of the asymptotic confidence interval for $\tau = \log(1-\alpha_2) - \log(1-\alpha_1)$, see (16). The arguments are $\hat{\alpha}_1$, k_1, n_1, $\hat{\alpha}_2$, k_2, n_2 and γ.

```
tau.aci <- function(R1,nitem1,nsample1,R2,nitem2,nsample2,clevel) {
 pcrit <- 0.5*(1-clevel)
 zcritl <- qnorm(pcrit,lower.tail=TRUE)
 zcritr <- qnorm(pcrit,lower.tail=FALSE)
 ww1 <- 2*nitem1/(nsample1*(nitem1-1))
 ww2 <- 2*nitem2/(nsample2*(nitem2-1))
 w <- sqrt(ww1+ww2)
 G <- log((1-R2)/(1-R1))
 acil <- G-zcritr*w
 acir <- G-zcritl*w
 list(lower=acil,upper=acir)
}
```

The function `tau.xci` computes the lower bound τ_L and upper bound τ_R of the exact confidence interval for $\tau = \log(1-\alpha_2) - \log(1-\alpha_1)$, see (16). The arguments are $\hat{\alpha}_1$, k_1, n_1, $\hat{\alpha}_2$, k_2, n_2 and γ.

```
tau.xci <- function(R1,nitem1,nsample1,R2,nitem2,nsample2,clevel,m) {
 pcrit <- 0.5*(1-clevel)
 qcrit <- 1-pcrit
 sim <- sort(G.random(m,nitem2,nsample2,nitem1,nsample1))
 Gcritl <- 0.5*(sim[floor(m*pcrit)]+sim[ceiling(m*pcrit)])
 Gcritr <- 0.5*(sim[floor(m*qcrit)]+sim[ceiling(m*qcrit)])
 G <- log((1-R2)/(1-R1))
 xcil <- Gcritr-G
 xcir <- Gcritl-G
 list(lower=xcil,upper=xcir)
}
```

The function `alphah.random` uses the representation (5) to generate a vector of m random values of $\hat{\alpha}$. The arguments are m, α, k and n.

```
alphah.random <- function(m,alpha,nitem,nsample) {
 df1 <- nsample*(nitem-1)
 df2 <- nsample
 1-(1-alpha)*rf(m,df1,df2)
}
```

The function `G.random` uses the representation (14) to generate a vector of m random values of G. The arguments are m, k_1, n_1, k_2 and n_2.

```
G.random <- function(m,nitem1,nsample1,nitem2,nsample2) {
 df11 <- nsample1*(nitem1-1)
```

```
  df12 <- nsample1
  df21 <- nsample2*(nitem2-1)
  df22 <- nsample2
  log(rf(m,df11,df12))-log(rf(m,df21,df22))
}
```

Table 1

Percentage points of G, for $k_1 = 2$ and $n_1 = 100$.

		$\gamma = 0.95$		$\gamma = 0.95$		$\gamma = 0.95$	
k_2	n_2	G_L	G_R	G_L	G_R	G_L	G_R
2	50	-0.5748	0.5737	-0.6856	0.6841	-0.9018	0.9021
	100	-0.4686	0.4669	-0.5589	0.5565	-0.7337	0.7330
	200	-0.4041	0.4042	-0.4823	0.4822	-0.6345	0.6367
4	50	-0.4881	0.5234	-0.5828	0.6241	-0.7656	0.8207
	100	-0.4180	0.4349	-0.4991	0.5187	-0.6573	0.6823
	200	-0.3779	0.3853	-0.4506	0.4591	-0.5928	0.6046
6	50	-0.4701	0.5120	-0.5613	0.6101	-0.7389	0.8032
	100	-0.4078	0.4289	-0.4868	0.5109	-0.6420	0.6700
	200	-0.3723	0.3812	-0.4441	0.4545	-0.5855	0.5989

Table 2

Percentage points of G, for $k_1 = 4$ and $n_1 = 100$.

		$\gamma = 0.95$		$\gamma = 0.95$		$\gamma = 0.95$	
k_2	n_2	G_L	G_R	G_L	G_R	G_L	G_R
2	50	-0.5487	0.5332	-0.6533	0.6381	-0.8589	0.8422
	100	-0.4358	0.4185	-0.5190	0.5004	-0.6818	0.6594
	200	-0.3653	0.3477	-0.4357	0.4146	-0.5733	0.5444
4	50	-0.4577	0.4790	-0.5451	0.5724	-0.7160	0.7537
	100	-0.3812	0.3809	-0.4546	0.4550	-0.5983	0.5985
	200	-0.3353	0.3248	-0.3998	0.3868	-0.5264	0.5074
6	50	-0.4380	0.4675	-0.5211	0.5573	-0.6853	0.7376
	100	-0.3703	0.3747	-0.4415	0.4462	-0.5808	0.5865
	200	-0.3290	0.3191	-0.3930	0.3802	-0.5175	0.5008

Table 3

Percentage points of G, for $k_1 = 6$ and $n_1 = 100$.

		$\gamma = 0.95$		$\gamma = 0.95$		$\gamma = 0.95$	
k_2	n_2	G_L	G_R	G_L	G_R	G_L	G_R
2	50	-0.5443	0.5249	-0.6492	0.6279	-0.8562	0.8297
	100	-0.4281	0.4087	-0.5095	0.4875	-0.6684	0.6408
	200	-0.3572	0.3366	-0.4257	0.4010	-0.5610	0.5265
4	50	-0.4521	0.4708	-0.5385	0.5617	-0.7060	0.7414
	100	-0.3732	0.3709	-0.4442	0.4414	-0.5833	0.5821
	200	-0.3259	0.3126	-0.3888	0.3727	-0.5128	0.4887
6	50	-0.4323	0.4575	-0.5148	0.5467	-0.6737	0.7229
	100	-0.3623	0.3625	-0.4319	0.4318	-0.5683	0.5705
	200	-0.3191	0.3073	-0.3811	0.3666	-0.5025	0.4832

Table 4

95% Confidence bounds for Cronbach's coefficient α, given that its estimator $\hat{\alpha}$ takes the value 0.6

		ID		*Proposed*		*Proposed*	
		Asymptotic bounds		*Asymptotic bounds*		*Exact bounds*	
n	k	*Lower*	*Upper*	*Lower*	*Upper*	*Lower*	*Upper*
50	2	0.3783	0.8217	0.3037	0.7702	0.2992	0.7717
	4	0.4189	0.7811	0.3710	0.7456	0.3848	0.7532
	6	0.4282	0.7718	0.3855	0.7396	0.4022	0.7487
100	2	0.4432	0.7568	0.4080	0.7297	0.4067	0.7303
	4	0.4720	0.7280	0.4491	0.7096	0.4555	0.7137
	6	0.4785	0.7215	0.4581	0.7047	0.4658	0.7097
200	2	0.4891	0.7109	0.4722	0.6968	0.4718	0.6971
	4	0.5095	0.6905	0.4984	0.6810	0.5014	0.6832
	6	0.5141	0.6859	0.5042	0.6773	0.5078	0.6800

Table 5

95% Confidence bounds for Cronbach's coefficient α, given that its estimator $\hat{\alpha}$ takes the value 0.7

		ID		*Proposed*		*Proposed*	
		Asymptotic bounds		*Asymptotic bounds*		*Exact bounds*	
n	k	*Lower*	*Upper*	*Lower*	*Upper*	*Lower*	*Upper*
50	2	0.5337	0.8663	0.4778	0.8277	0.4744	0.8288
	4	0.5642	0.8358	0.5283	0.8092	0.5386	0.8149
	6	0.5712	0.8288	0.5391	0.8047	0.5516	0.8115
100	2	0.5824	0.8176	0.5560	0.7973	0.5550	0.7977
	4	0.6040	0.7960	0.5868	0.7822	0.5916	0.7853
	6	0.6089	0.7911	0.5936	0.7786	0.5993	0.7823
200	2	0.6168	0.7832	0.6042	0.7726	0.6039	0.7728
	4	0.6321	0.7679	0.6238	0.7608	0.6261	0.7624
	6	0.6356	0.7644	0.6282	0.7580	0.6309	0.7600

Table 6

95% Confidence bounds for Cronbach's coefficient α, given that its estimator $\hat{\alpha}$ takes the value 0.8

		ID		*Proposed*		*Proposed*	
		Asymptotic bounds		*Asymptotic bounds*		*Exact bounds*	
n	k	*Lower*	*Upper*	*Lower*	*Upper*	*Lower*	*Upper*
50	2	0.6891	0.9109	0.6518	0.8851	0.6496	0.8858
	4	0.7095	0.8905	0.6855	0.8728	0.6924	0.8766
	6	0.7141	0.8859	0.6927	0.8698	0.7011	0.8744
100	2	0.7216	0.8784	0.7040	0.8649	0.7033	0.8652
	4	0.7360	0.8640	0.7246	0.8548	0.7277	0.8569
	6	0.7393	0.8607	0.7290	0.8524	0.7329	0.8549
200	2	0.7446	0.8554	0.7361	0.8484	0.7359	0.8485
	4	0.7547	0.8453	0.7492	0.8405	0.7507	0.8416
	6	0.7571	0.8429	0.7521	0.8386	0.7539	0.8400

Table 7

95% Confidence bounds for Cronbach's coefficient α, given that its estimator $\hat{\alpha}$ takes the value 0.9

		ID		*Proposed*		*Proposed*	
		Asymptotic bounds		*Asymptotic bounds*		*Exact bounds*	
n	k	*Lower*	*Upper*	*Lower*	*Upper*	*Lower*	*Upper*
50	2	0.8446	0.9554	0.8259	0.9426	0.8248	0.9429
	4	0.8547	0.9453	0.8428	0.9364	0.8462	0.9383
	6	0.8571	0.9429	0.8464	0.9349	0.8505	0.9372
100	2	0.8608	0.9392	0.8520	0.9324	0.8517	0.9326
	4	0.8680	0.9320	0.8623	0.9274	0.8639	0.9284
	6	0.8696	0.9304	0.8645	0.9262	0.8664	0.9274
200	2	0.8723	0.9277	0.8681	0.9242	0.8680	0.9243
	4	0.8774	0.9226	0.8746	0.9203	0.8754	0.9208
	6	0.8785	0.9215	0.8761	0.9193	0.8770	0.9200

Table 8

Simulated nominal coverage of confidence bounds for Cronbach's coefficient, for $\alpha = 0.6$, $\gamma = 0.95$ and $m = 1000000$

n	k	*ID*	*Proposed asymptotic*	*Proposed exact*
50	2	0.936090	0.947198	0.949839
	4	0.947737	0.947275	0.949793
	6	0.949625	0.946946	0.950016
100	2	0.943031	0.948515	0.949760
	4	0.948857	0.948626	0.949817
	6	0.949864	0.948383	0.949853
200	2	0.946771	0.949647	0.950294
	4	0.949407	0.949021	0.949674
	6	0.949877	0.949202	0.949880

Table 9

Simulated nominal coverage of confidence bounds for Cronbach's coefficient, for $\alpha = 0.7$, $\gamma = 0.95$ and $m = 1000000$

n	k	*ID*	*Proposed asymptotic*	*Proposed exact*
50	2	0.936591	0.947551	0.950162
	4	0.947779	0.947558	0.950036
	6	0.949616	0.947638	0.950170
100	2	0.943381	0.948784	0.950040
	4	0.948878	0.948872	0.949910
	6	0.949929	0.948603	0.950136
200	2	0.946555	0.949397	0.950050
	4	0.949627	0.949437	0.950118
	6	0.949689	0.949071	0.949703

Table 10

Simulated nominal coverage of confidence bounds for Cronbach's coefficient, for $\alpha = 0.8$, $\gamma = 0.95$ and $m = 1000000$

n	k	*ID*	*Proposed asymptotic*	*Proposed exact*
50	2	0.936454	0.947304	0.949892
	4	0.947517	0.947521	0.949816
	6	0.950001	0.947494	0.950279
100	2	0.943408	0.948710	0.950105
	4	0.948548	0.948351	0.949498
	6	0.949769	0.948179	0.949617
200	2	0.946416	0.949229	0.949938
	4	0.949023	0.949150	0.949759
	6	0.949752	0.949058	0.949881

Table 11

Simulated nominal coverage of confidence bounds for Cronbach's coefficient, for $\alpha = 0.9$, $\gamma = 0.95$ and $m = 1000000$

			Proposed	*Proposed*
n	k	*ID*	*asymptotic*	*exact*
50	2	0.936012	0.946971	0.949687
	4	0.947452	0.947068	0.949676
	6	0.949292	0.946881	0.949430
100	2	0.943171	0.948546	0.949857
	4	0.948488	0.948159	0.949487
	6	0.949843	0.948416	0.949917
200	2	0.946480	0.949235	0.949895
	4	0.949236	0.949174	0.949719
	6	0.949093	0.948529	0.949206

Table 12

95% Confidence bounds for comparing two Cronbach's coefficients, given that their estimators $\hat{\alpha}_1$ and $\hat{\alpha}_2$ take the values 0.7 and 0.8, respectively. Here δ denotes $\alpha_1 - \alpha_2$, and τ denotes $\ln\left(\frac{1-\alpha_2}{1-\alpha_1}\right)$.

			Asymptotic bounds for δ		*Asymptotic bounds for τ*		*Exact bounds for τ*	
n_1	n_2	k_2	*Lower*	*Upper*	*Lower*	*Upper*	*Lower*	*Upper*
2	50	2	-0.2616	0.0616	-1.0844	0.2735	1.0929	-0.2708
	50	4	-0.2413	0.0413	-0.9598	0.1489	0.9442	-0.1490
	50	6	-0.2300	0.0300	-0.8856	0.0746	0.8927	-0.0795
	100	2	-0.2484	0.0484	-1.0042	0.1933	1.0359	-0.1785
	100	4	-0.2339	0.0339	-0.9115	0.1006	0.9278	-0.1028
	100	6	-0.2260	0.0260	-0.8581	0.0472	0.8553	-0.0428
	200	2	-0.2456	0.0456	-0.9869	0.1760	1.0032	-0.1545
	200	4	-0.2324	0.0324	-0.9013	0.0904	0.9284	-0.0789
	200	6	-0.2252	0.0252	-0.8524	0.0415	0.8629	-0.0309

Table 13

95% Confidence bounds for comparing two Cronbach's coefficients, given that their estimators $\hat{\alpha}_1$ and $\hat{\alpha}_2$ take the values 0.7 and 0.8, respectively.

			Asymptotic bounds for δ		*Asymptotic bounds for τ*		*Exact bounds for τ*	
n_1	n_2	k_2	*Lower*	*Upper*	*Lower*	*Upper*	*Lower*	*Upper*
4	50	2	-0.2467	0.0467	-1.0456	0.2347	1.0479	-0.2452
	50	4	-0.2240	0.0240	-0.9115	0.1006	0.9144	-0.1113
	50	6	-0.2109	0.0109	-0.8289	0.0179	0.8031	-0.0380
	100	2	-0.2320	0.0320	-0.9598	0.1489	0.9769	-0.1352
	100	4	-0.2154	0.0154	-0.8581	0.0472	0.8508	-0.0469
	100	6	-0.2062	0.0062	-0.7975	-0.0135	0.7916	0.0126
	200	2	-0.2288	0.0288	-0.9410	0.1301	0.9611	-0.1122
	200	4	-0.2136	0.0136	-0.8466	0.0357	0.8475	-0.0326
	200	6	-0.2052	0.0052	-0.7909	-0.0201	0.7900	0.0176

Table 14

95% Confidence bounds for comparing two Cronbach's coefficients, given that their estimators $\hat{\alpha}_1$ and $\hat{\alpha}_2$ take the values 0.7 and 0.8, respectively.

			Asymptotic bounds for δ		*Asymptotic bounds for* τ		*Exact bounds for* τ	
n_1	n_2	k_2	*Lower*	*Upper*	*Lower*	*Upper*	*Lower*	*Upper*
6	50	2	-0.2435	0.0435	-1.0375	0.2266	1.0417	-0.2583
	50	4	-0.2202	0.0202	-0.9013	0.0904	0.8958	-0.1060
	50	6	-0.2066	0.0066	-0.8166	0.0057	0.7968	-0.0290
	100	2	-0.2284	0.0284	-0.9505	0.1396	0.9818	-0.1352
	100	4	-0.2113	0.0113	-0.8466	0.0357	0.8392	-0.0336
	100	6	-0.2017	0.0017	-0.7842	-0.0268	0.7770	0.0268
	200	2	-0.2252	0.0252	-0.9314	0.1204	0.9508	-0.1021
	200	4	-0.2095	0.0095	-0.8349	0.0239	0.8292	-0.0429
	200	6	-0.2007	0.0007	-0.7773	-0.0336	0.7792	0.0262

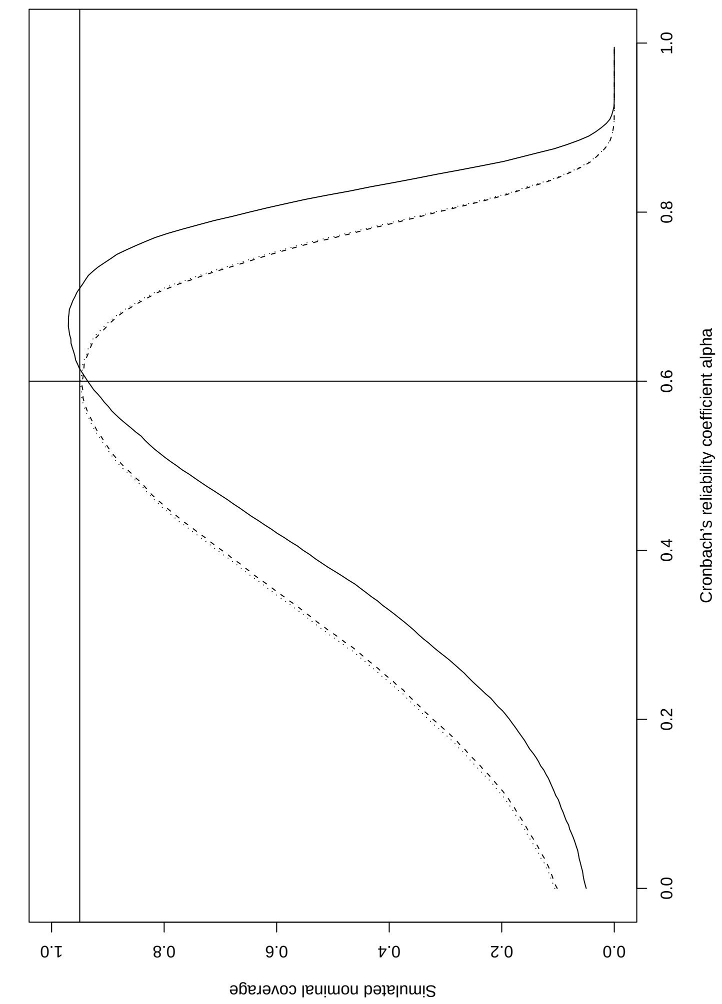

Fig. 1. Simulated nominal coverage of 95% confidence bounds for Cronbach's reliability coefficient α for a test with $k = 2$ items, administered to $n = 50$ subjects, when the "true" reliability of the test is 0.6. The solid, the dashed and the dotted lines indicate the coverage of the ID bounds, the proposed asymptotic bounds, and the proposed exact bounds. Observe that the dashed and dotted lines more or less coincide.

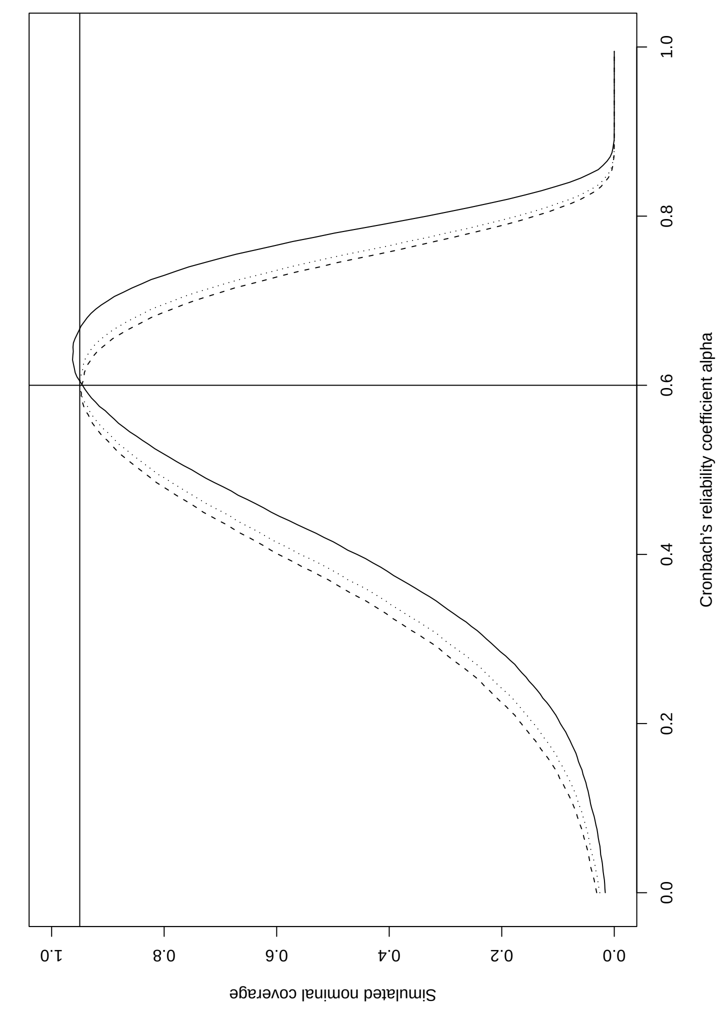

Fig. 2. Simulated nominal coverage of 95% confidence bounds for Cronbach's reliability coefficient α for a test with $k = 4$ items, administered to $n = 50$ subjects, when the "true" reliability of the test is 0.6. The solid, the dashed and the dotted lines indicate the coverage of the ID bounds, the proposed asymptotic bounds, and the proposed exact bounds.

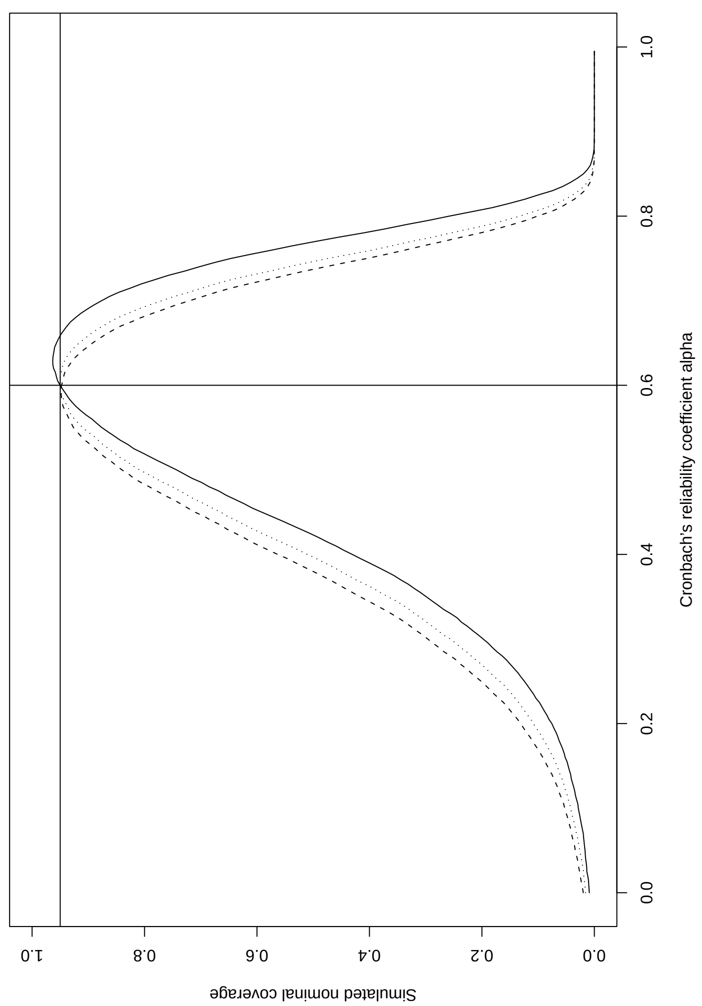

Fig. 3. Simulated nominal coverage of 95% confidence bounds for Cronbach's reliability coefficient α for a test with $k = 6$ items, administered to $n = 50$ subjects, when the "true" reliability of the test is 0.6. The solid, the dashed and the dotted lines indicate the coverage of the ID bounds, the proposed asymptotic bounds, and the proposed exact bounds.

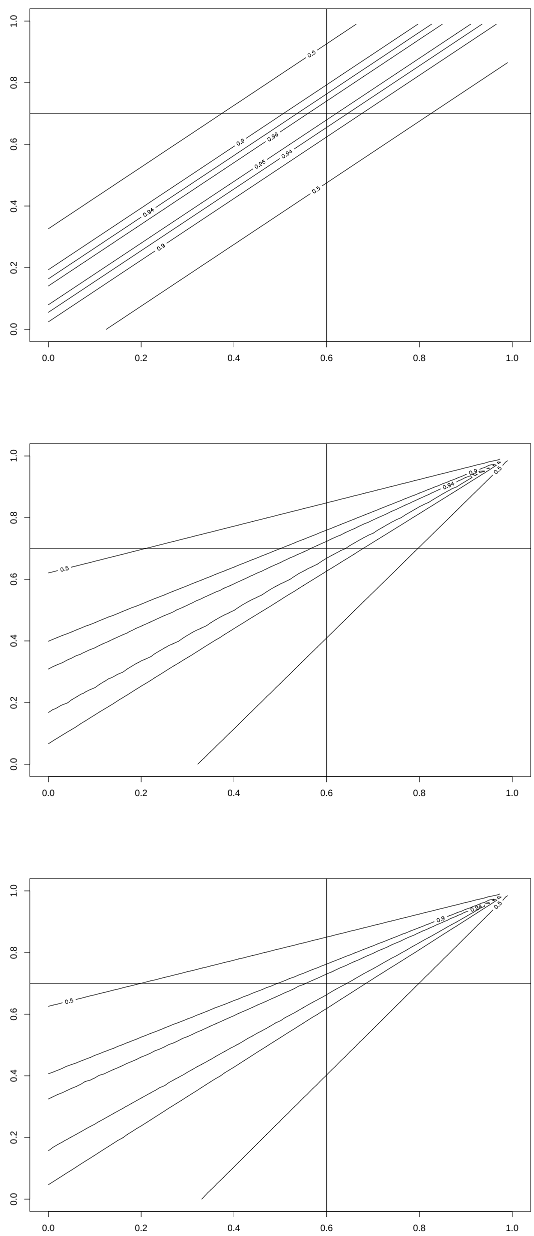

Fig. 4. Contour plot of simulated nominal coverage of 95% asymptotic confidence bounds, where the "true" $\alpha_1 = 0.6$, $k_1 = 2$, $n_1 = 100$, the "true" $\alpha_2 = 0.7$, $k_2 = 2$, $n_2 = 50$. Top: asymptotic confidence bounds for $\delta = \alpha_1 - \alpha_2$. Center: asymptotic confidence bounds for $\tau = \log(1 - \alpha_2) - \log(1 - \alpha_1)$. Bottom: exact confidence bounds for τ.

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