

If we're so different, why do we keep overlapping? When 1 plus 1 doesn't make 2

Rory Wolfe, James Hanley

In the last decade, guidelines for the presentation of statistical results in medical journals have emphasized confidence intervals (CIs) as an adjunct to, or even a replacement for, statistical tests and *p* values. Because of the intimate links between the 2 concepts, authors now use statements like “the 95% CI overlaps 0” where they would formerly have stated “the difference is not statistically significant at the 5% level.” Although this interchangeability is technically correct in 1-sample situations, it does not carry over fully to comparisons involving 2 samples. A frequently encountered misconception is that if 2 independent 95% CIs overlap each other, as they do in Fig. 1, then a statistical test of the difference will not be statistically significant at the 5% level.

Why is this not necessarily so? Consider the means in 2 independent groups, mean_A and mean_B , with for simplicity mean_A being the smaller of the 2. The 95% CI for the mean in group A is approximately given by mean_A plus or minus twice the standard error of the mean for that group, SE_A , and correspondingly for group B. A mathematical check for whether these CIs overlap is given by adding the distance 2SE_A (from mean_A to the upper bound of the CI) to 2SE_B and comparing this sum with the distance between the 2 means, that is, mean_B minus mean_A (Fig. 2). The CIs overlap when

$$[1] \quad \text{mean}_B - \text{mean}_A < 2\text{SE}_A + 2\text{SE}_B$$

But overlapping confidence intervals do not demonstrate that group means are not statistically significantly different from each other. In a 2-sample *t*-test to compare 2 means, significance is attained at the 0.05 level if the *t* statistic exceeds the critical value of about 2, which occurs when the difference between the means exceeds twice its standard error, namely, if

$$[2] \quad \text{mean}_B - \text{mean}_A > 2\sqrt{(\text{SE}_A^2 + \text{SE}_B^2)}$$

This standard error reflects the fact that the standard error of a difference involves summing the standard error of each estimate, but doing so by “adding in quadrature,” for example,

$$[3] \quad 1 \text{ “+” } 1 = \sqrt{1^2 + 1^2} = 1.414$$

Thus, to evaluate the overlap of 2 95% CIs and to determine whether at the same time the difference between the

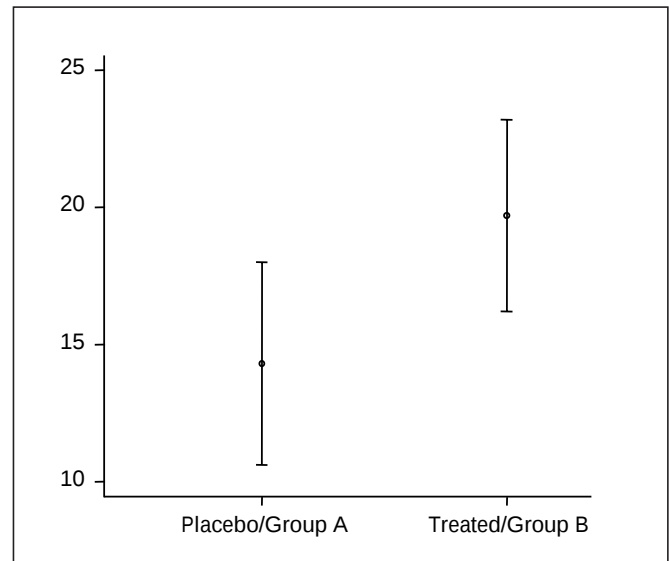


Fig. 1: Group means with confidence intervals that overlap.

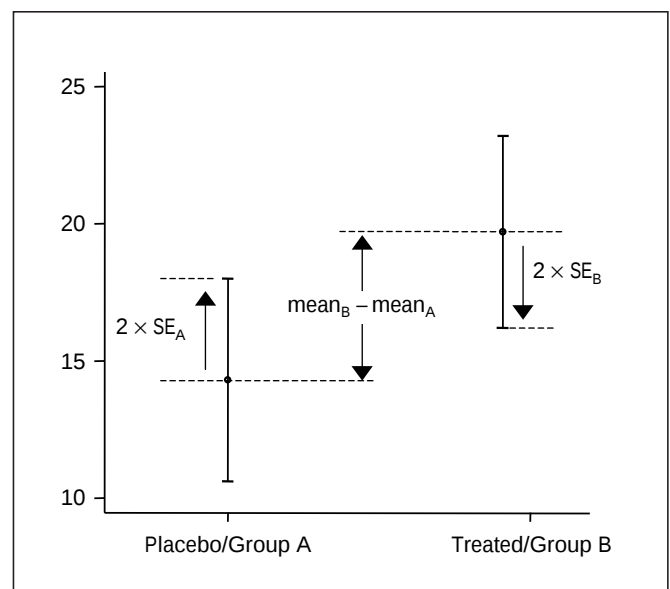


Fig. 2: Confidence intervals and comparison of 2 group means (hypothetical clinical trial data: $\text{SE}_A = \text{SE}_B = 1.8$, means differ by 3 SE; assuming $n > 30$ and independent samples, the 2-sided *p* value for testing the difference in means is approximately 0.036). SE = standard error of the mean.

means is significant at the 0.05 level, the following rough rule can be used:

$$[4] \quad 2\sqrt{(\text{SE}_A^2 + \text{SE}_B^2)} < \text{mean}_B - \text{mean}_A < 2\text{SE}_A + 2\text{SE}_B$$

If SE_A and SE_B are equal, the condition is as follows:

$$[5] \quad 2.83 \text{ SE} < \text{mean}_B - \text{mean}_A < 4 \text{ SE}$$

When one SE is 25% larger than the other, the boundaries are 3.2 and 4.5 times the smaller SE. As the lower boundary remains close to 3, Moses¹ was prompted to display group means with error bars that were 1.5 SE around the mean in order to have a “by eye” test of significance between the 2 group means while presenting the information in the 2 groups separately.

This article has been peer reviewed.

Dr. Wolfe is with the Department of Epidemiology and Preventive Medicine, Monash University, Melbourne, Victoria, Australia. Dr. Hanley is with the Department of Epidemiology and Biostatistics, McGill University, Montreal, Que.

Competing interests: None declared.

Contributors: Both authors independently conceived of the material for this article. Both were involved in writing the article, and both have seen and approved the final version.

Reference

1. Moses LE. Graphical methods in statistical analysis. *Annu Rev Public Health* 1987;8:309-53.

Correspondence to: Dr. Rory Wolfe, Department of Epidemiology and Preventive Medicine, Central and Eastern Clinical School, Monash University and the Alfred Hospital, Commercial Rd., Prahran, Victoria 3183, Australia; fax 61 3 9903 0556; rory.wolfe@med.monash.edu.au

BOOKS RECEIVED

Amnesty International. **Ethical codes and declarations relevant to the health professions.** London: Amnesty International; 2000. 173 pp. US \$7 (paper)

Carrick P. **Medical ethics in the ancient world.** Washington: Georgetown University Press; 2001. 266 pp. US \$60 (cloth) ISBN 0-87840-848-7 \$27.50 (paper) ISBN 0-87840-849-5

Eadie MJ, Bladin PF. **A disease once sacred: a history of the medical understanding of epilepsy.** England: John Libbey & Company Limited; 2001. 248 pp. US \$39 (paper) ISBN 0-86196-607-4

Fenster JM. **Ether day.** Scarborough (ON): Harper Collins Publishers; 2001. 278 pp. \$36.50 (cloth) ISBN 0-06-019523-1

Finley GA, McGrath PJ, editors. **Acute and procedure pain in infants and children.** Seattle (WA): IASP Press; 2001. 183 pp. US \$70 (cloth) ISBN 0-931092-39-6

Fredriksson EH, editor. **A century of science publishing: a collection of essays.** Amsterdam: IOS Press; 2001. 312 pp. ISBN 1-58603-148-1

Fulop N, Allen P, Clarke A, Black N, editors. **Studying the organization and delivery of health services: research methods.** New York: Taylor & Francis Books Ltd.; 2001. 228 pp. US \$16.99 (paper) ISBN 0-415-25763-8

Kushner TK, Thomasma DC, editors. **Ward ethics: dilemmas for medical students and doctors in training.** Cambridge (MA): Cambridge University Press; 2001. 265 pp. US \$85 (cloth) ISBN 0-521-80291-1 \$29.95 (paper) ISBN 0-521-66452-7

Liddle PF. **Disordered mind and brain: the neural basis of mental symptoms.** London: The Royal College of Psychiatrists; 2001. 301 pp. £40 (cloth) ISBN 1-901242-65-X

Osler W. **Osler's "A way of life" & other addresses with commentary & annotations.** Durham (NC): Duke

University Press; 2001. 378 pp. US \$29.95 (cloth) ISBN 0-8223-2682-5

Pfäfflin M, Fraser R, Thorbecke R, Specht U, Wolf P, editors. **Comprehensive care for people with epilepsy.** England: John Libbey & Company Limited; 2001. 365 pp. US \$96 (cloth) ISBN 0-86196-610-4

Porter R.. **Bodies politic: disease, death and doctors in Britain 1650-1900.** Ithaca (NY): Cornell University Press; 2001. 328 pp. US \$35 (cloth) ISBN 0-8014-3953-1

Sabat SR. **The experience of Alzheimer's disease: life through a tangled veil.** Maldon (MA): Blackwell Publishers Inc.; 2001. 361 pp. US \$26.95 (paper) ISBN 0-631-21666-9

Wilson WR, Sande MA. **Current diagnosis & treatment in infectious diseases.** New York: McGraw-Hill Medical Publishing Division; 2001. 985 pp. US \$69.95 (paper) ISBN 0-8385-1494-4 (domestic) 0-07-118185-3 (international)